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Electron Inertia Induce Nonlinear Acoustic Modes Near the Transonic Region in a Multispecies Plasma

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Abstract: In presence of multicomponent ionic plasma, more than one plasma sound waves are possible depending on the number of ions. In this work, we considered the presence of a negative ion along with the normal positive ion. Here we show that finite but weak electron inertial delay effect causes a resonant excitation of the ion acoustic solitons near the transonic zone, which depends on $m_e/(m_{+i1,2})$ ratio. It has been seen that under such situation near the plasma sheath transonic zone, the KdV equations have complex coefficients. We have shown that even in presence of the complex coefficients soliton like solution can be derived only for infinitely long wavelength source perturbation. It is seen that when the negative ions mass becomes equal to that of a dust particle then similar excitation situation as in case of colloidal plasma can be retrieved. It is plausible that such kind of a resonant excitation may lead to acoustic turbulence near the plasma sheath edge. A detailed discussion of the nonlinear acoustic mode analyses in the transonic regime with negative ionic impurity using a hydrodynamic approach is presented.

Keywords: Electron inertia, Transonic zone, Sheath edge, Nonlinear acoustic modes, Soliton, Multispecies plasma.

I. INTRODUCTION

The area of plasma sheath has been of active interest for the scientist and engineers for its wide scale application in various branches of science and engineering [1 -3]. The physics of plasma sheath formation for normal two-component plasma was put forwarded long back by Tonks and Langmuir [4] in 1929. The condition for sheath formation requires that the ions drift speed should have velocity greater than the ion sound speed i.e. $C_s = \sqrt{kT_e/M}$ (k is the Boltzmann constant, T_e is the electron temperature and M is the ionic mass), at the sheath edge was given by David Bohm 1949 [5]. The problem of plasma sheath transition from quasi-neutral bulk plasma to the non-neutral and nonlinear plasma sheath at the plasma wall boundary has been of great attention due to the presence of singularity at the plasma sheath edge in a two scale model as pointed by Riemman [6]. In the two scale theory of presheath-sheath transition singularity arises due to transition from a long scale (of the order of the half of the system length) to the sheath scale of the order of few times of Debye length. To overcome this patching problem Riemann et. al. suggested a new concept of three scale transition region with an introduction of an intermediate scale between the presheath and the sheath having an intermediate length [7]. It is noteworthy to mention here that the one of the early pioneering work with an analytical model in this regard was proposed by Self [8] in 1963. He tried to arrive at an exact solution for the Tonks and Langmuir equation [4] for the plasma and sheath by using a series expansion method of the integral equation and solved it numerically. Most of these theoretical models are self sufficient for particular situations and no universal consensus was arrived.

There has been a significant work about the issue of singularity by various workers under different force field configuration and compositions [9 – 13]. In spite of numerous and rigorous work done by various scientist, still this area beholds with large resource for newer problems. The first author along with his other co-workers put forwarded a new concept of the formation of the sheath of the basis of wave turbulence model [14]. Though the concept is still in its infancy but it gives a new dimension to the already going works about the plasma sheath transition problem.

An important aspect of all these theoretical models is that the electrons are always considered to be inertialess compared to that of the ions because of the ions much heavier mass than that of the electrons, i.e. $me /mi \rightarrow 0$. But in the last few years this concept has come under critical review [15 -21]. Such an assumption can hold good only for a cold plasma ($T_i << T_e$) and for non drifting ions i.e. for stationary ions. In case of drifting ions, when we analyse the propagation of the ion acoustic wave in a moving frame of reference near the sheath entrance, which we can call as the transonic zone also, the universal consideration of the Boltzmannian distribution of the electrons do not hold good. It has been shown that when weak but finite electron inertial effect is considered than ion acoustic wave fluctuations of a two-component plasma system with drifting ions reveals a new mode of instability [16]. Such type of instabilities can be called as resonance mode instability, which will take place only when the plasma flow velocity exceeds the ion acoustic speed of non-drifting plasmas. Such situations can be seen to exist in the transonic region of the plasma sheath system and also in solar and other stellar wind plasmas. In fact, under such a situation the Debye sheath condition formation also gets modified [19]. Under such assumption the collective degrees of freedom will be modified.

Thus, it is quite easily realizable that the nonlinear mode of the ion acoustic wave in the transonic zone in a bounded plasma system also gets modified [15]. The modification of the nonlinear mode of the ion acoustic wave has been shown in a two component plasma system along with the presence of colloidal particles [18]. The investigation of solitary wave propagation in plasmas has gained lot of momentum since the discovery of ion acoustic soliton by Washimi and Taniuti [22] described by the Korteweg–de Vries (KdV) equation. The laboratory verification of the existence of acoustic solitons came four years later when Ikezi et al. [23] showed its existence in two component plasma system. There has been some effort to investigate the effect of high energy drifting ions for propagation of arbitrary-amplitude solitary waves in plasma with and without the consideration of the inertial effect of the electrons. Varieties of physical situations of drifting ions of high energy with [24, 25] and without inclusion of the electron inertia motion have been considered for theoretical description of the ion acoustic soliton. Similar situations are practically realizable in the magnetoshperic region of the earth. But in these studies the relativistic effect of the electrons were considered. In these works it was mentioned that the existence of complex coefficients of the KdV equation as a condition for non existence of the ion acoustic solitons. That's why the linearly unstable condition in the velocity regime was excluded. But interestingly such condition hides some other interesting physical facts and condition for non existence of solitary waves were refuted, which is reported in the literature as mentioned in Ref. [16].

In this work, we propose to see a similar effect with multi-component species. The existence of plasma with multi-component is naturally found in the magnetospheric region and the Van-Allen belts of the earth. Moreover, number of experiments related to the ion acoustic waves in a multi-component and colloidal or dust plasma has been carried out in the laboratory as reported in the literatures. But the ion acoustic wave propagation in a drifting ion beam system near the transonic zone is yet to be done. Here we will present a theoretical study of the instability in the ion acoustic fluctuations in a drifting ion beam system in presence of two different ionic species (with positive charge) due to weak but finite electron inertial delay effect. Moreover the importance of solitons has wide range applications in communications and other fields [26-28].

This paper is basically divided into four major sections including the introduction. In the next section we shall elaborately present the physical model of the problem along with the detailed mathematical formulations. In the third section we shall cover detail discussions about the results derived in the previous section. Finally in the last section the conclusion of the work along with the scope for future direction will be presented.

II. PHYSICAL MODEL AND MATHEMATICAL FORMULATIONS

In this work we shall show that a driven KdV equation can be derived for the steady-state behavior of the nonlinear normal mode of an acoustic wave under linearly unstable conditions of the spectral component as shown in [16]. We have considered a simple unmagnetized plasma consisting of three components with two positive ions of different having nearly equal masses, same degree of ionization but of different density along with electrons. The system is considered to be collisionless plasma in which the ions are drifting with uniform velocity at near supersonic speed. Further no extra sink or source terms are considered. The linear acoustic waves are resonantly excited due to weak but finite electron inertial delay effect. However, we made another assumption that the transonic zone near the plasma

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sheath edge has a finite extension. The primary excitation mechanism is based on the compressibility of the electrons, which otherwise becomes redundant Boltzmannian distribution for the electrons is considered. Moreover it is assumed that the plasma wall absorbs the plasma through surface recombination and the neutral particles get re-circulated into the bulk plasma. Under the hydrodynamic model the basic governing equations for the electron, the two ions and are discussed below.

The electron continuity equation is given below

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0. \tag{1}$$

And the electron momentum equation is

$$M_e\left(\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial x}\right) = -n_e eE - KT_e\frac{\partial n_e}{\partial x}. \tag{2}$$

The ion continuity equation for first ion is given below

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial}{\partial x}(n_{i1} v_{i1}) = 0. \tag{3}$$

Similarly the momentum equation for first ion is

$$M_{i1} n_{i1}\left(\frac{\partial v_{i1}}{\partial t} + v_{i1}\frac{\partial v_{i1}}{\partial x}\right) = n_{i1} eE, \tag{4}$$

and the continuity equation for second ion is

$$\frac{\partial n_{i2}}{\partial t} + \frac{\partial}{\partial x}(n_{i2} v_{i2}) = 0. \tag{5}$$

Similarly the momentum equation for second ion is

$$M_{i2} n_{i2}\left(\frac{\partial v_{i2}}{\partial t} + v_{i2}\frac{\partial v_{i2}}{\partial x}\right) = n_{i2} eE. \tag{6}$$

And finally the poison's for the plasma system is

$$\frac{\partial^2 \emptyset}{\partial x^2} = \frac{e}{\epsilon_0}(n_e - n_{i1} - n_{i2}). \tag{7}$$

Where n_e is the density, v_e is the velocity and M_e, is the mass and T_e is the temperature of the electrons. Similarly, n_{i1} and n_{i2} are the densities, v_{i1} and v_{i2} are velocities and M_{i1} and M_{i2} are the masses of the two ions respectively. Φ is the electrostatic potential. We shall rewrite the equations now onwards in their normalized form. The potential will be normalized by the thermal potential kT_e/e, space coordinate electron Debye length λ_{De}, time by ion plasma oscillation frequency of the ion having mass M_{i1}, which is c_{s1}/λ_{De}, where c_{s1} is the acoustic velocity of ion having mass M_{i1}. Again the electron density is normalized with equilibrium plasma density $n_{o.}$ The zeroth order i.e. the leading order solution of the electron density can be expressed as the usual Boltzmann distribution given below:

$$n_e = e^{(\emptyset)}. \tag{8}$$

The above equation can be derived using the usual electron momentum equation (9) in the limit of $1/\varepsilon_m \to 0$, where the electron continuity equation becomes redundant. The lowest order correction in the electron density solution (due to finite but weak electron inertia) as mentioned in Ref. [16] is given below.

$$\frac{\partial e^{\emptyset}}{\partial t} + \frac{\partial}{\partial x}\left(e^{\emptyset}\vartheta_e\right) = 0. \tag{9}$$

Finally, the lowest order correction of electron inertia motion in the electron density distribution is obtained by coupling of electron fluid velocity (from continuity equation (1)) along with the the electron inertial effect in the full electron momentum equation is given below.

$$\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial x} = \varepsilon_m\frac{\partial \emptyset}{\partial x} - \frac{\varepsilon_m}{n_e}\frac{\partial n_e}{\partial x}. \tag{10}$$

Here $\varepsilon_m = M_{i1}/M_e$, M_{i1} is the mass of first ion. Now eqs. (8) and (9) constitute a set of two self consistently coupled equations to describe the linear and nonlinear behaviors of the ion acoustic mode. A detail justification of the unusual nature of Eq. (8) and about the use of Eqs. (8) and (9) for linear and nonlinear acoustic wave description in plasmas with drifting ions is explained in one of the author's earlier work [16].

The ion response in acoustic perturbations is described by the traditional full inertial equation of the drifting ions. The ion continuity equation for the first ion dealing with ion density flux conservation is given as,

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial}{\partial x}(n_{i1} v_{i1}) = 0\ . \tag{11}$$

Furthermore, the ion momentum equation in the cold limit is given as follows

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$$\frac{\partial v_{i1}}{\partial t} + v_{i1}\frac{\partial v_{i1}}{\partial x} = -\frac{\partial \emptyset}{\partial x}. \quad (12)$$

Similarly for the second ion we will have

$$\frac{\partial n_{i2}}{\partial t} + \frac{\partial}{\partial x}(n_{i2}v_{i2}) = 0\,, \quad (13)$$

$$\frac{\partial v_{i2}}{\partial t} + v_{i2}\frac{\partial v_{i2}}{\partial x} = -\frac{\partial \emptyset}{\partial x}. \quad (14)$$

Finally, Poisson's equation describing the potential distribution is given below,

$$\frac{\partial^2 \emptyset}{\partial x^2} = (n_e - n_{i1} - n_{i2}). \quad (15)$$

Stretching coordinates as given below are used to carry out the mathematical calculations are to transform the independent space and time coordinates.

$$\xi = \epsilon^{1/2}(x - Mt), \quad (16)$$

$$\tau = \epsilon^{3/2}t. \quad (17)$$

Where, ξ and τ are the transformed space and time coordinates in moving frame and ε is a small dimensionless parameter signifying the strength of nonlinearity and dispersion. M is the Mach number, which is normally defined as the normalized phase velocity of the normal mode of the nonlinear acoustic wave tin a moving frame. Using the stretched variables the derivatives w.r.t. time and space can be written as follows:

$$\frac{\partial}{\partial t} = \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau}.$$

So using the above equation we have

$$\frac{\partial}{\partial t} = -M\varepsilon^{1/2}\frac{\partial}{\partial \xi} + \varepsilon^{3/2}\frac{\partial}{\partial \tau}\,, \quad (18)$$

The transformed space derivative is

$$\frac{\partial}{\partial x} = \varepsilon^{1/2}\frac{\partial}{\partial \xi}. \quad (19)$$

Using linear perturbation technique the dependent variables are expended as follows.

$$n_{e,i1,i2} = 1 + \epsilon n^{(1)}_{e,i1,i2} + \epsilon^2 \epsilon n^{(2)}_{e,i1,i2} + \cdots,$$

$$v_e = \epsilon v_e^{(1)} + \epsilon^2 v_e^{(2)} + \cdots,$$

$$v_{i1,i2} = 1 + \epsilon v^{(1)}_{i1,i2} + \epsilon^2 v^{(2)}_{i1,i2} + \cdots,$$

$$\emptyset = \epsilon \emptyset^{(1)} + \epsilon^2 \emptyset^{(2)} + \cdots,$$

Here we have considered the average (unperturbed) velocity of electron to be zero and ions has finite drift velocity. The order by order analysis of the usual electron continuity and momentum equations in their normalized form are given below:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0\,, \quad (20)$$

$$\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial x} = \varepsilon_m\frac{\partial \emptyset}{\partial x} - \frac{\varepsilon_m}{n_e}\frac{\partial n_e}{\partial x}\,. \quad (21)$$

Using the same normalization condition as discussed earlier the order by order analysis of these two eqs. (20) and (21) yields the following linear and nonlinear equations of first and second order, respectively.

From continuity equation we will have

$$\epsilon^{3/2} \to -M\frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\partial v_e^{(1)}}{\partial \xi} = 0\,, \quad (22)$$

$$\epsilon^{5/2} \to \frac{\partial v_e^{(2)}}{\partial \xi} = -\frac{\partial n_e^{(1)}}{\partial \xi} - \frac{\partial}{\partial \xi}\left(n_e^{(1)}v_e^{(1)}\right) + M\frac{\partial n_e^{(2)}}{\partial \xi}. \quad (23)$$

From electron momentum equation we will get

$$\varepsilon^{3/2} \to -M\frac{\partial v_e^{(1)}}{\partial \xi} = \varepsilon_m\frac{\partial \emptyset^{(1)}}{\partial \xi} - \varepsilon_m\frac{\partial n_e^{(1)}}{\partial \xi}\,, \quad (24)$$

$$\varepsilon^{5/2} \to \frac{\partial n_e^{(2)}}{\partial \xi} = -\frac{1}{\epsilon_m}\frac{\partial v_e^{(1)}}{\partial \tau} - \frac{1}{\epsilon_m}v_e^{(1)}\frac{\partial v_e^{(1)}}{\partial \xi} + n_e^{(1)}\frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\partial \emptyset^{(2)}}{\partial \xi} + \frac{M}{\epsilon_m}\frac{\partial v_e^{(2)}}{\partial \xi}. \quad (25)$$

On integrating the linear order equation (22) & (24) will yield

$$Mn_e^{(1)} = v_e^{(1)}\,, \quad (26)$$

$$n_e^{(1)} = \emptyset^{(1)} + \frac{M}{\epsilon_m}v_e^{(1)}\,. \quad (27)$$

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From the above equation it is quite evident the normal Boltzmannian electron distribution can be recovered from equation (27) in the limit of $1/\varepsilon_m$ tending to zero, i.e. when the electrons are inertialess. Under such condition the use of electron continuity equation becomes needless. So it becomes evident that the electron continuity equation is required only when finite electron inertia is considered. The inclusion of the electron continuity equation becomes meaningful only when weak but finite electron inertia is considered. This is introduced as a higher order equation to introduce correction in leading order electron density solution obtained by the usual Boltzmann relation. Accordingly, by substituting the leading order linear solution of electron density $\emptyset^{(1)} = n_e^{(1)}$ in equation (26), we determine the lowest order correction in electron fluid velocity as

$$v_e^{(1)} = M\emptyset^{(1)}\,. \tag{28}$$

The corresponding modification in the leading order electron density solution due to weak but finite electron inertia can be introduced by incorporating the electron inertial term of equation (27), which will yield

$$n_e^{(1)} = \left(1+\frac{M^2}{\varepsilon_m}\right)\emptyset^{(1)}\,. \tag{29}$$

Again if we directly solve (26) and (27), we will find

$$n_e^{(1)} = \left(1-\frac{M^2}{\varepsilon_m}\right)^{-1}\emptyset^{(1)}\,, \tag{30}$$

and

$$v_e^{(1)} = M\left(1-\frac{M^2}{\varepsilon_m}\right)^{-1}\emptyset^{(1)}\,. \tag{31}$$

The weak but finite electron inertia correction can be introduced by binomial expansion of the multiplying factor$\Big(1 -$ $M2\varepsilon m-1$ ~1+$M2\varepsilon m$. This will lead to correction in the electron fluid velocity because of the appearance of electron fluid velocity in the inertial term of the electron momentum equation as well as in continuity equation, which may be mentioned as the consistent manner of introducing the correction due to inertial effect.

Now, using the well-known reductive perturbation method we find the dynamic response of the nonlinear normal mode of the acoustic fluctuations under unstable conditions. After that an order by order analysis of equation (8) to (14) is carried out to deduce the following equations of linear and nonlinear orders.

First the electron continuity equation (8) is analysed, which will give

$$\left(-M\varepsilon^{1/2}\frac{\partial}{\partial\xi}+\varepsilon^{3/2}\frac{\partial}{\partial\tau}\right)\left(e^{\epsilon\emptyset^{(1)}+\varepsilon^2\emptyset^2}\right)+\varepsilon^{1/2}\frac{\partial}{\partial\xi}\left(\left[e^{\varepsilon\emptyset^{(1)}+\varepsilon^2\emptyset^{(2)}+\cdots}\right]\left[\varepsilon v_e^{(1)}+\varepsilon^2 v_e^{(2)}+\cdots\right]\right).$$

We get the following form of the linear and nonlinear equations

$$\epsilon^{3/2} \to -M\frac{\partial\emptyset^{(1)}}{\partial\xi}+\frac{\partial v_e^{(1)}}{\partial\xi}=0 \tag{32}$$

and

$$\epsilon^{5/2} \to \frac{\partial v_e^{(2)}}{\partial\xi} = -\frac{\partial n_e^{(1)}}{\partial\xi}-\frac{\partial}{\partial\xi}\left(n_e^{(1)}v_e^{(1)}\right)+M\frac{\partial n_e^{(2)}}{\partial\xi}+M\frac{\partial\emptyset^{(2)}}{\partial\xi}\,. \tag{33}$$

And from the electron momentum equation we have

$$\begin{aligned}\left(-M\varepsilon^{1/2}\frac{\partial}{\partial\xi}+\varepsilon^{3/2}\frac{\partial}{\partial\tau}\right)\left[\varepsilon v_e^{(1)}+\varepsilon^2 v_e^{(2)}+\cdots\right]+\left[\varepsilon v_e^{(1)}+\varepsilon^2 v_e^{(2)}+\cdots\right]\\ +\varepsilon^{1/2}\frac{\partial}{\partial\xi}\left[\varepsilon v_e^{(1)}+\varepsilon^2 v_e^{(2)}+\cdots\right]\\ =\epsilon_m\varepsilon^{1/2}\frac{\partial}{\partial\xi}\left(\epsilon\emptyset^{(1)}+\epsilon^2\emptyset^{(2)}+\cdots\right)\\ -\frac{\epsilon_m}{\left(1+\epsilon n_e^{(1)}+\epsilon^2 n_e^{(2)}\right)}\varepsilon^{1/2}\frac{\partial}{\partial\xi}\left(1+\epsilon n_e^{(1)}+\epsilon^2 n_e^{(2)}\right).\end{aligned}$$

The linear and nonlinear term can be separated out as shown below.

$$\varepsilon^{3/2} \to -M\frac{\partial v_e^{(1)}}{\partial\xi}=\varepsilon_m\frac{\partial\emptyset^{(1)}}{\partial\xi}-\varepsilon_m\frac{\partial n_e^{(1)}}{\partial\xi}\,, \tag{34}$$

$$\varepsilon^{5/2} \to \frac{\partial n_e^{(2)}}{\partial\xi}=-\frac{1}{\varepsilon_m}\frac{\partial v_e^{(1)}}{\partial\tau}-\frac{1}{\epsilon_m}v_e^{(1)}\frac{\partial v_e^{(1)}}{\partial\xi}+n_e^{(1)}\frac{\partial n_e^{(1)}}{\partial\xi}+\frac{\partial\emptyset^{(2)}}{\partial\xi}+\frac{M}{\epsilon_m}\frac{\partial v_e^{(2)}}{\partial\xi} \tag{35}$$

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Similarly from continuity equation for the first ion and doing a order by order analysis we get the following linear and nonlinear terms.

$$\varepsilon^{3/2} \rightarrow (M - v_{i10})\frac{\partial n_{i1}^{(1)}}{\partial \xi} = \frac{\partial v_{i1}^{(1)}}{\partial \xi}, \tag{36}$$

$$\varepsilon^{5/2} \rightarrow \frac{\partial n_{i1}^{(1)}}{\partial \tau} - (M - v_{i10})\frac{\partial n_{i1}^{(2)}}{\partial \xi} + \frac{\partial v_{i1}^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(n_{i1}^{(1)} v_{i1}^{(1)}\right) = 0. \tag{37}$$

From the momentum equation of the first ion and following the same procedure we have,

$$\varepsilon^{3/2} \rightarrow -M\frac{\partial v_{i1}^{(1)}}{\partial \xi} + v_{i1o}\frac{\partial v_{i1}^{(1)}}{\partial \xi} = -\frac{\partial \emptyset^{(1)}}{\partial \xi}\ , \tag{38}$$

$$\varepsilon^{5/2} \rightarrow \frac{\partial v_{i1}^{(2)}}{\partial \xi} = \frac{1}{(M - v_{i10})}\frac{\partial v_{i1}^{(1)}}{\partial \tau} + \frac{1}{(M - v_{i10})} v_{i1}^{(1)}\frac{\partial v_{i1}^{(1)}}{\partial \xi} + \frac{1}{(M - v_{i1o})}\frac{\partial \emptyset^{(2)}}{\partial \xi}. \tag{39}$$

Similarly from continuity equation for second ion and carrying out order by order analysis we get the following linear and nonlinear terms.

$$\varepsilon^{3/2} \rightarrow (M - v_{i20})\frac{\partial n_{i2}^{(1)}}{\partial \xi} = \frac{\partial v_{i2}^{(1)}}{\partial \xi}, \tag{40}$$

and

$$\varepsilon^{5/2} \rightarrow \frac{\partial n_{i2}^{(1)}}{\partial \tau} - (M - v_{i20})\frac{\partial n_{i2}^{(2)}}{\partial \xi} + \frac{\partial v_{i2}^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(n_{i2}^{(1)} v_{i2}^{(1)}\right) = 0\ . \tag{41}$$

From the momentum equation of second ion following linear and nonlinear order equations are obtained.

$$\varepsilon^{3/2} \rightarrow -M\frac{\partial v_{i2}^{(1)}}{\partial \xi} + v_{i2o}\frac{\partial v_{i2}^{(1)}}{\partial \xi} = -\frac{\partial \emptyset^{(1)}}{\partial \xi}, \tag{42}$$

and

$$\varepsilon^{5/2} \rightarrow \frac{\partial v_{i2}^{(2)}}{\partial \xi} = \frac{1}{(M - v_{i20})}\frac{\partial v_{i2}^{(1)}}{\partial \tau} + \frac{1}{(M - v_{i20})} v_{i2}^{(1)}\frac{\partial v_{i2}^{(1)}}{\partial \xi} + \frac{1}{(M - v_{i2o})}\frac{\partial \emptyset^{(2)}}{\partial \xi}. \tag{43}$$

Finally from the poisson's equation we have,

$$\varepsilon \rightarrow n_e^{(1)} = n_{i1}^{(1)} + n_{i2}^{(1)}\ , \tag{44}$$

and

$$\epsilon^2 \ \rightarrow \frac{\partial^2 \emptyset^{(1)}}{\partial \xi^2} = n_e^{(2)} - n_{i1}^{(2)} - n_{i2}^{(2)}. \tag{45}$$

Taking the space derivative of equation (32) we have

$$\frac{\partial^3 \emptyset^{(1)}}{\partial \xi^3} = \frac{\partial n_e^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi}\left(n_{i1}^{(2)} + n_{i2}^{(2)}\right). \tag{46}$$

Then integrating equation (32) & (34) we have

$$v_e^{(1)} = M\emptyset^{(1)}\ , \tag{47}$$

and

$$-Mv_e^{(1)} = \varepsilon_m \emptyset^{(1)} - \varepsilon_m n_e^{(1)}. \tag{48}$$

Using (47) in (48) we get

$$n_e^{(1)} = \left(1 + \frac{M^2}{\varepsilon_m}\right)\emptyset^{(1)}, \tag{49}$$

Again integrating equation (36) and (40) we get

$$n_{i1}^{(1)} = \frac{v_{i1}^{(1)}}{(M - v_{i10})}\ , \tag{50}$$

$$n_{i2}^{(1)} = \frac{v_{i2}^{(1)}}{(M - v_{i20})}\ . \tag{51}$$

Integrating (8) we get

$$-Mv_{i1}^{(1)} + v_{i10} v_{i1}^{(1)} = -\emptyset^{(1)}. \tag{52}$$

Simplifying the above equation we will get

$$v_{i1}^{(1)} = \frac{\emptyset^{(1)}}{(M - v_{i10})}\ . \tag{53}$$

And similarly integrating (42) we have

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$$v_{i2}^{(1)} = \frac{\emptyset^{(1)}}{(M-v_{i20})} \quad . \tag{54}$$

Now substituting (47) and (49) in (35) will give us the following equations.

$$\frac{\partial n_e^{(2)}}{\partial \xi} = -\frac{1}{\varepsilon_m}\frac{\partial v_e^{(1)}}{\partial \tau} - \frac{1}{\epsilon_m} v_e^{(1)} \frac{\partial v_e^{(1)}}{\partial \xi} + n_e^{(1)}\frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\partial \emptyset^{(2)}}{\partial \xi} + \frac{M}{\epsilon_m}\frac{\partial v_e^{(2)}}{\partial \xi}$$

$$\frac{\partial n_e^{(2)}}{\partial \xi} = -\frac{M}{\varepsilon_m}\frac{\partial \emptyset^{(1)}}{\partial \tau} - \frac{M^2}{\varepsilon_m}\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \frac{\partial \emptyset^{(2)}}{\partial \xi} + \left(1+\frac{M^2}{\varepsilon_m}\right)^2 \emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \frac{M}{\varepsilon_m}\frac{\partial v_e^{(2)}}{\partial \xi}$$

Now replacing $\frac{\partial v_e^{(2)}}{\partial \xi}$ in above equation from (33) we and on simplification we get

$$\frac{\partial n_e^{(2)}}{\partial \xi} = -\frac{M}{\varepsilon_m}\frac{\partial \emptyset^{(1)}}{\partial \tau} + \left(1+\frac{M^4}{\varepsilon_m^2}\right)\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \left(1+\frac{M^2}{\varepsilon_m}\right)\frac{\partial \emptyset^{(2)}}{\partial \xi}. \tag{55}$$

Now using linear order solution (50) & (53) in equation (39) which is

$$\frac{\partial v_{i1}^{(2)}}{\partial \xi} = \frac{1}{(M-v_{i10})}\frac{\partial v_{i1}^{(1)}}{\partial \tau} + \frac{1}{(M-v_{i10})} v_{i1}^{(1)}\frac{\partial v_{i1}^{(1)}}{\partial \xi} + \frac{1}{(M-v_{i1o})}\frac{\partial \emptyset^{(2)}}{\partial \xi}.$$

This gets reduced to

$$\frac{\partial n_{i1}^{(2)}}{\partial \xi} = \frac{2}{(M-v_{i10})^3}\frac{\partial \emptyset^{(1)}}{\partial \tau} + \frac{3}{(M-v_{i10})^4}\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \frac{1}{(M-v_{i10})^2}\frac{\partial \emptyset^{(2)}}{\partial \xi}. \tag{56}$$

Using linear order solution (51) and (54) in equation (43) will give

$$\varepsilon^{5/2} \to \frac{\partial v_{i2}^{(2)}}{\partial \xi} = \frac{1}{(M-v_{i20})}\frac{\partial v_{i2}^{(1)}}{\partial \tau} + \frac{1}{(M-v_{i20})} v_{i2}^{(1)}\frac{\partial v_{i2}^{(1)}}{\partial \xi} + \frac{1}{(M-v_{i2o})}\frac{\partial \emptyset^{(2)}}{\partial \xi}.$$

This gets reduced to

$$\frac{\partial n_{i2}^{(2)}}{\partial \xi} = \frac{2}{(M-v_{i20})^3}\frac{\partial \emptyset^{(1)}}{\partial \tau} + \frac{3}{(M-v_{i20})^4}\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \frac{1}{(M-v_{i20})^2}\frac{\partial \emptyset^{(2)}}{\partial \xi} \quad . \tag{57}$$

Now using (47), (49), (50), (51), (53), (54) in (44) we get

$$\left(1+\frac{M^2}{\varepsilon_m}\right)\emptyset^{(1)} = \frac{v_{i1}^{(1)}}{(M-v_{i10})} + \frac{v_{i2}^{(1)}}{(M-v_{i20})}.$$

Now replacing $v_{i1}^{(1)}$ and $v_{i2}^{(1)}$ from (53) and (54) we get the following relation

$$\left(1+\frac{M^2}{\varepsilon_m}\right)\emptyset^{(1)} = \frac{\emptyset^{(1)}}{(M-v_{i10})^2} + \frac{\emptyset^{(1)}}{(M-v_{i20})^2}.$$

$$1+\frac{M^2}{\varepsilon_m} = \frac{1}{(M-v_{i10})^2} + \frac{1}{(M-v_{i20})^2}. \tag{58}$$

The relation (58) is the dispersion relation of linear acoustic waves associated with the considered plasma system. Similarly using (55), (56), (57) in (46) we will get

$$\frac{\partial^3 \emptyset^{(1)}}{\partial \xi^3} = \left(\frac{-2}{\varepsilon_m} - \frac{2}{(M-v_{i10})^3} - \frac{2}{(M-v_{i20})^3}\right)\frac{\partial \emptyset^{(1)}}{\partial \tau} + \left[\left(1+\frac{M^4}{\varepsilon_m^2}\right) - \frac{3}{(M-v_{i10})^4} - \frac{3}{(M-v_{i20})^4}\right]\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi}$$
$$+\left[\left(1+\frac{M^2}{\varepsilon_m}\right) - \frac{1}{(M-v_{i10})^2} - \frac{1}{(M-v_{i20})^2}\right]\frac{\partial \emptyset^{(2)}}{\partial \xi}.$$

Now the coefficient of third term vanishes from dispersion relation (58) and rearranging and dividing by 2 we get

$$\left(\frac{M}{\varepsilon_m} + \frac{1}{(M-v_{i10})^3} + \frac{1}{(M-v_{i20})^3}\right)\frac{\partial \emptyset^{(1)}}{\partial \tau} +$$
$$+\frac{1}{2}\left[-\left(1+\frac{M^4}{\varepsilon_m^2}\right) + \frac{3}{(M-v_{i10})^4} + \frac{3}{(M-v_{i20})^4}\right]\emptyset^{(1)}\frac{\partial \emptyset^{(1)}}{\partial \xi} + \frac{1}{2}\frac{\partial \emptyset^3}{\partial \xi^3} = 0\,. \tag{59}$$

Now this above equation is the desired KdV equation for the nonlinear acoustic mode. Again we define Doppler shifted Mach number M_{D1} and M_{D2} as below.

$$M_{D1} = M - v_{i10}$$
$$M_{D2} = M - v_{i20}$$

Using the Doppler shifted Mach number in derived dispersion relation can be rewritten as;

$$(M_{D1} + v_{i10})^2 = \epsilon_m\left(\frac{1}{M_{D1}^2} + \frac{1}{M_{D1}^2} - 1\right) \tag{60}$$

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Here we have assumed $M_{D1} \sim M_{D2}$ since we have considered their masses to be nearly same. As discussed in a reference [16], the linear resonant instability condition $k.v_{i10} < 0$, defines a resonant Mach value as $M_{D1} \sim v_{i10} \geq 1.41$. Using this condition in Eq. 60 we can solve the linear dispersion to yield the following Mach solution:

$$M_{D1} + v_{i10} = i\sqrt{\varepsilon_m}\sqrt[4]{2} \times \left(\sqrt{2} - v_{i10}\right) \quad . \tag{61}$$

It is obvious that the Mach number becomes a complex quantity indicating the possibility of instability. Now, the real and imaginary parts of the Doppler shifted Mach number can be written as follows $M_{Dr1} = v_{i10} \sim 1.41$. It is clearly noticeable that the coefficients in the derived KdV equation (58) are complex. According to previous authors' arguments [25], the complex nature of these coefficients prevents the formation of a soliton as a nonlinear normal mode solution of the acoustic wave. However, we will show that the complex nature of the coefficients could be accommodated in global phase modification of the amplitude and Mach no. of the usual KdV soliton.

Let us first consider the coefficient of the temporal differential term which can be simplified in the form of $A+iB$ where A and B are calculated as below

$$A = \frac{M_r}{\varepsilon_m} + \frac{M_{Dr1}^3 - 3M_{Dr1}M_{Di1}^2}{(M_{Dr1}^2 + M_{Di1}^2)^3} + \frac{M_{Dr2}^3 - 3M_{Dr2}M_{Di2}^2}{(M_{Dr2}^2 + M_{Di2}^2)^3}$$

$$B = \frac{M_{Di1}^3 + 3M_{Di1}M_{Dr1}^2}{(M_{Dr1}^2 + M_{Di1}^2)^3} + \frac{M_{Di2}^3 - 3M_{Di2}M_{Dr2}^2}{(M_{Dr2}^2 + M_{Di2}^2)^3} + \frac{M_{Di1}}{\varepsilon_m}$$

Similarly the non linear term can be expressed in the form of $C+iD$, where

$$C = 1 + \frac{(M_{Dr1}^2 - M_{Di1}^2)^2 - 4M_{Dr1}^2M_{Di1}^2}{\epsilon_m^2} - \frac{3((M_{Dr1}^2 - M_{Di1}^2)^2 - 4M_{Dr1}^2M_{Di1}^2)}{(M_{Dr1}^2 - M_{Di1}^2)^4} - \frac{3((M_{Dr2}^2 - M_{Di2}^2)^2 - 4M_{Dr2}^2M_{Di2}^2)}{(M_{Dr2}^2 - M_{Di2}^2)^4}$$

$$D = \frac{4(M_{Dr1}^2 - M_{Di1}^2)M_{Dr1}^2M_{Di1}^2}{\epsilon_m^2} + \frac{12(M_{Dr1}^2 - M_{Di1}^2)M_{Dr1}^2M_{Di1}^2}{(M_{Dr1}^2 - M_{Di1}^2)^4} + \frac{12(M_{Dr2}^2 - M_{Di2}^2)M_{Dr2}^2M_{Di2}^2}{(M_{Dr2}^2 - M_{Di2}^2)^4}$$

It is mathematically justified that the complex coefficients $A+iB$ and $C+iD$ becomes implicit function of time by virtue of instability and can be represented as an exponential function like $K_0e^{i\theta(t)}$ and $M_0e^{i\varphi(t)}$. Hence, the new real coefficient can now be defined as $K_0=(A^2+B^2)^{1/2}$ and $M_0=(C^2+D^2)^{1/2}$ respectively. The linear growth effect in the usual soliton amplitude can be included by considering the following transformation of the nonlinear wave potential amplitude defined as $\emptyset^{(1)} = \emptyset_m(\xi,\tau)e^{-\gamma t}$ where γ represent the normalized growth rate [16]. This can be introduced as a kind of global temporal phase transformation. Considering this, $\emptyset^{(1)}$ should now be replaced by $\emptyset_m(\xi,\tau)e^{-\Upsilon t}$ in all terms of equation (59). After that the first term becomes,

$$K_0e^{\theta(t)}e^{-\gamma\tau}\left(\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau} - 1\right),$$

and second term gives

$$M_0e^{\varphi(t)}e^{-2i\gamma\tau}\emptyset_m\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau}.$$

And the dispersive term becomes

$$\frac{\partial^3\emptyset_m(\xi,\tau)}{\partial\xi^3}e^{-\gamma\tau}.$$

Now the equation (59) can be rewritten in the following form

$$K_0e^{\theta(t)}e^{-\gamma\tau}\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau} + M_0e^{\varphi(t)}e^{-2i\gamma\tau}\emptyset_m\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau} + \frac{1}{2}\frac{\partial^3\emptyset_m(\xi,\tau)}{\partial\xi^3}e^{-\gamma\tau} = \gamma K_0e^{\theta(t)}\emptyset_m \tag{62}$$

Now the consideration of the temporal arguments discussed above as $\theta(t) \sim \varphi(t)$ proportional to $\gamma\tau$ reduces the exponential term to unity so the equation (62) becomes

$$K_0\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau} + M_0\emptyset_m\frac{\partial\emptyset_m(\xi,\tau)}{\partial\tau} + \frac{1}{2}\frac{\partial^3\emptyset_m(\xi,\tau)}{\partial\xi^3} = \gamma K_0\emptyset_m \tag{63}$$

This is the required dynamical equation to describe the evolution of the nonlinear normal mode behavior of the finite but weak amplitude of acoustic fluctuations under unstable conditions of the plasma system under consideration. This equation may be again written in form of the standard KdV equation for a steady state condition by introducing the

transformations, η=ξ-τ. Using this transformation of coordinate we will have $\frac{\partial}{\partial\xi}=\frac{\partial}{\partial\eta}$ and $\frac{\partial}{\partial\tau}=-\frac{\partial}{\partial\eta}$.
Henceforth, eq. (63) can be written in the following manner,

$$\frac{1}{2}\frac{\partial^3\phi_m}{\partial\eta^3}+M_0\phi_m\frac{\partial\phi_m}{\partial\eta}-K_0\frac{\partial\phi_m}{\partial\eta}=\gamma K_0\phi_m. \quad (64)$$

Equation (64) is the steady state form of the KdV equation with a source term on the R.H.S. Such an equation can be further solved numerically.

III. RESULTS AND DISCUSSIONS

The above theoretical calculation is carried out for a special case of occurrence of multispecies plasma with drifting ions. We have analyzed the behavior of the nonlinear normal mode of the ion acoustic wave near the transonic zone signified by the condition of $k.v_{i10}\leq 0$. This is the condition of the resonant instability for the fluctuations to grow in a moving beam. In general, the unstable behavior of the propagation of soliton is not considered to exist under normal circumstances. In literatures as mentioned above, it has been mentioned that the existence of complex coefficients in the KdV equation should be a condition for non-existence of solitons. But contradictory to such assumptions we have shown with our theoretical work that solitary waves can still exist even though the coefficients may be complex. Under such condition it gives rise to a special situation in which we observed that the KdV equation has got a source term. This source term infact acts as the driving source for the instability.

It is noteworthy to mention here that the modified form of the KdV equation has been derived by invoking the idea of global phase modification. With this new idea we have observed that nonlinear solution of the normal acoustic mode is possible, which may gives rise to solitary kind of waves. As discussed in the experimental section in Ref. [16], it is also possible that oscillatory shock like solutions may be possible under a steady state condition in the transonic zone. A more appropriate comparison can be given by doing a numerical integration of Eq. (64), which is similar to the results mentioned in Ref. [16]. The numerical integration has been left out as a future course of the work. The form of the driven KdV equation derived for our case of multispecies plasma is structurally same as that derived by the authors in Ref. [16]. We can reasonably defend that as soon as the solitary wave passes through the unstable transonic zone near the plasma sheath edge, it may experience transient phase modifications, which leads to the formation of an oscillatory shock. Under such a driving mechanism of the instability we can argue that there may be adiabatic rearrangement of the spectral components of the usual solitary wave.

Now if we analyze the formation of such a driven KdV equation, which is possibly going to give rise to oscillatory shock like solitary wave solutions as mentioned in Ref. [16], the whole genesis lies in the inclusion of the finite but weak electron inertial effect. Henceforth, we can say that the linear growth of the instability is related to finite but weak electron inertial delay effect that is supposed to be active in the transonic zone. Since the form is same, we may expect the similar kind of numerical solution as derived for two-component plasma [16] system, where it was found that normal soliton structure was not retained and oscillatory solution was obtained. This theory can be extended by increasing the mass ratio and different Mach number. This work becomes a general description for a multi-component plasma system and can be easily extend to study for dust particles by changing the mass ratio. Numerical investigation of equation (64) or a complete simulation of equation (63) will give us more indepth physics about evolution of such acoustic waves. Further, this analysis gives us the idea that the plasma sheath edge is a rich zone where various wave activities are possible. Hence, there is a possibility of wave-wave coupling and particle-wave coupling. Under such condition wave turbulence activities cannot be denied, since to activate turbulence there must be some kind of instability in the system. So, it is permissible to hypothesize the idea of a wave turbulence model to describe the sheath formation, which should give a new dimension to the ongoing problem of the plasma sheath edge singularity.

IV. CONCLUSION

From the detailed analytical work about the investigation of the propagation behaviour of the nonlinear normal mode of the ion acoustic wave near the transonic zone of the plasma sheath in a multi-component plasma system, indicates that finite but weak electron inertia can act as a source for driving the acoustic mode unstable. The linear growth of such instability shows that solitary shock like solutions are possible inspite of the coefficients of the derived KdV equation becoming complex. We have justified with our analytical work that some new physics can be derived

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from such a situation. This type of instabilities can be well justified to be of the nature of resonant type instabilities, which leads to a global phase modification of the spectral components of the soliton. This will be wise to mention that more detailed information of the nature of propagation could have been arrived at by doing a complete simulation of the new KdV equation with the modified coefficients and the source term. Such kind of studies should help us to understand many such phenomena in the laboratory system with multispecies plasma or in the Earth's Van Allen belt or the auroral regions. Moreover we would like to suggest that the plasma sheath edge region should be a rich region where different wave activities are likely to occur leading to an exchange of energy between the wave and the particles, such that the ions can derive energy from the waves to enter the sheath region.

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