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δ(δg)*-Closed sets in Topological Spaces

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ABSTRACT: In this paper we introduce and study a new class of generalised closed sets called $\delta(\delta g)^*$-closed sets in topological spaces using δg-closed sets. Moreover we analyse the relations between $\delta(\delta g)^*$-closed sets and already existing various closed sets. It is independent of δg-closed sets and weaker than δg*-closed sets. The class of $\delta(\delta g)^*$-closed sets is properly placed between the classes of δg*-closed sets and $\delta g^{\#}$ -closed sets and a chain of relations is proved as follows.

r-closed $\longrightarrow$ π-closed $\longrightarrow$ δ-closed $\longrightarrow$ δg*-closed $\longrightarrow$ δ(δg)*-closed $\longrightarrow$ $\delta g^{\#}$-closed $\longrightarrow$ gδ-closed

KEYWORDS:.g-closed sets, δg-closed sets , δg*-closed sets and $\delta(\delta g)^*$-closed sets.

I. INTRODUCTION

The concept of generalised closed (briefly, g-closed) sets were introduced and investigated by Norman Levine [2] in 1970. Velicko [3] introduced δ-open sets which are stronger than open sets in 1968. By combining the concepts of δ-closedness and g-closedness,Julian Dontchev [4] proposed a class of generalised closed sets called δg-closed set in 1996.Sudha.R and Sivakamasundari.K [20] introduced and investigated a new concept of generalised closed sets namely δg^*-closed sets in 2012.The aim of this paper is to introduce a new class of generalised closed sets called $\delta(\delta g)^*$-closed sets.

In this paper in section II we give some preliminaries which are used to carry out our work.In section III we introduce the definition of $\delta(\delta g)^*$-closed sets and analyse the relations between $\delta(\delta g)^*$-closed sets and δ-closed sets, δg^*-closed sets,gδ-closed,rg-closed,gpr-closed, $\delta g^{\#}$closed, rwg-closed, gspr-closed, πg-closed, πgp-closed, πgsp-closed, g-closed, αg-closed, *g-closed, $\alpha\hat{g}$-closed, #gs-closed, δg-closed, $\delta\hat{g}$-closed and gp-closed, g*p-closed, g*s-closed and (gs)*-closed sets. Throughout this paper (X, τ) represents a non empty topological space on which no separation axioms are mentioned unless otherwise specified.

II.PRELIMINARIES

Definition 2.1. A subset A of (X, τ) is called a
1) Regular open set if A = int(cl(A))
2) Semi open set if A⊆ cl(int(A)).
3) α-open set if A⊆ int(cl(int(A))) .
4) Pre-open set if A⊆ int(cl(A)).
5) Semi pre open set if A⊆cl(int(cl(A)))
6) π-open set if it is the finite union of regular open sets.
7) δ-open set if it is the union of regular open sets.

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The complements of the above mentioned sets are called regular closed, semi closed, α- closed, pre- closed, Semi pre closed, π-closed and δ-closed sets respectively.
The intersection of all regular closed(resp.semi-closed ,α-closed, pre-closed, semi pre-closed, π-closed and δ-closed) subsets of (X, τ) containing A is called the regular closure(resp.semi-closure, α-closure, pre- closure, semi pre closure, π-closure, and δ-closure) of A and is denoted by rcl(A) (resp. scl(A) αcl(A), pcl(A), spcl(A), πcl(A), and δcl(A)).

Definition 2.2 A subset A of a topological space (X, τ) is called

1) **generalized closed** (briefly **g-closed**) [2] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
2) **regular generalized closed** (briefly **rg-closed**) [5] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is regular open in (X, τ).
3) **α-generalized closed** (briefly **αg-closed**) [6] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
4) **generalized pre-closed** (briefly **gp-closed**) [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
5) **generalized pre regular closed(**briefly **gpr-closed**) [8] if pcl(A)$\subseteq$U whenever A$\subseteq$U,U is regular open in (X, τ).
6) **δ-generalized closed** (briefly **δg-closed**) [4] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
7) **generalized δ-closed** (briefly **gδ-closed**) [9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δ-open in (X, τ).
8) **$\delta g^{\#}$-closed** [10] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δ-open in (X, τ).
9) **$\alpha\hat{g}$-closed** [11] ifαcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$-open in (X, τ).
10) **$\delta\hat{g}$-closed** [12] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$-open in (X, τ).
11) **$g^{*}p$-closed** [13] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is g-open in (X, τ).
12) **$*g$-closed** [14] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$-open in (X, τ).
13) **#gs-closed** [14] if scl(A) $\subseteq$ U whenever A $\subseteq$ U, U is *g-open in (X, τ).
14) **g*s-closed** [15] if scl(A) $\subseteq$ U whenever A $\subseteq$ U, U is gs-open in (X, τ).
15) **regular weakly generalised closed(**briefly**,rwg-closed)** [16] if cl(int(A)) $\subseteq$U whenever A $\subseteq$ U, U is regular open in (X, τ).
16) **generalised semi pre regular closed**(briefly,**gspr-closed**) [17] if spcl(A) $\subseteq$U whenever A$\subseteq$U, U is regular open in (X, τ).
17) **π-generalised closed**(briefly,**πg-closed**) [18] if cl(A) $\subseteq$U whenever A $\subseteq$ U, U is π-open in (X, τ).
18) **π-generalised pre- closed**(briefly,**πgp-closed**) [19] if pcl(A) $\subseteq$U whenever A $\subseteq$ U, U is π-open in (X, τ).
19) **π-generalised semi pre-closed**(briefly,**πgsp-closed**) [17] if spcl(A) $\subseteq$U whenever A$\subseteq$U,U is π-open in (X,τ).
20) **δ-generalised star closed**(briefly, **δg^{*}-closed**) [20] if δcl(A) $\subseteq$U whenever A $\subseteq$ U, U is g- open in (X, τ).
21) **$(gs)^{*}$-closed set** [21] if cl(A) $\subseteq$U whenever A $\subseteq$ U, U is gs-open in (X, τ).

Remark 2.3.
r-closed(open) $\rightarrow$$\pi$-closed(open) $\rightarrow$ δ-closed(open) $\rightarrow$ δg*-closed(open) $\rightarrow$ δg-closed(open) $\rightarrow$g-closed(open).

Remark 2.4. For every subset A of X, spcl(A) $\subseteq$pcl(A) $\subseteq$ δcl(A). (Proposition 3.2 of [22])

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III. $\delta(\delta g)^*$-CLOSED SETS

Definition 3.1. A subset A of a topological space (X, τ) is said to be **$\delta(\delta g)^*$-closed set** if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$, U is δg-open in (X, τ).
The class of all $\delta(\delta g)^*$-closed sets of (X, τ) is denoted by $\delta(\delta G)^*C(X, \tau)$.

Proposition 3.2. Every δ-closed set is $\delta(\delta g)^*$-closed but not conversely.

Proof: Let A be a δ- closed set and U be any δg-open set containing A. Since A is δ-closed, δcl(A)=A. Therefore $\delta cl(A)=A \subseteq U$ and hence A is $\delta(\delta g)^*$ -closed.
Counter Example 3.3 Let X={a,b,c}, τ={ϕ,X,{a},{a,b}}. In this topology the subset {c} is $\delta(\delta g)^*$-closed but not δ-closed.

Proposition 3.4 Every δg^*-closed set is $\delta(\delta g)^*$-closed but not conversely.

Proof: Let A be δg^*-closed and U be any δg- open set containing A in X. By remark 2.3. every δg-open set is g-open and A is δg^*-closed, $\delta cl(A) \subseteq U$. Hence A is $\delta(\delta g)^*$- closed.
Counter Example 3.5 Let X={a,b,c},τ={ϕ,X,{a,b}}, Then the subset {b,c} is $\delta(\delta g)^*$-closed but not δg^*- closed in (X, τ).

Proposition 3.6 Every $\delta(\delta g)^*$-closed set is gδ- closed but not conversely.

Proof: Let A be $\delta(\delta g)^*$-closed set and U be any δ- open set containing A in X. By remark 2.3. every δ-open is δg-open and A is $\delta(\delta g)^*$-closed, $\delta cl(A) \subseteq U$. For every subset A of X, $cl(A) \subseteq \delta cl(A)$ and so $cl(A) \subseteq U$ and hence A is gδ-closed.
Counter Example 3.7 Let X={a,b,c}, τ={ϕ,X,{a},{a,b},{a,c}}. Then the subset {a} is gδ-closed but not $\delta(\delta g)^*$- closed in (X, τ).

Proposition 3.8 Every $\delta(\delta g)^*$-closed set is rg-closed but not conversely.

Proof: Let A be $\delta(\delta g)^*$-closed and U be any regular open set containing A in X. By remark 2.3. every regular open is δg- open and A is $\delta(\delta g)^*$ -closed, $\delta cl(A) \subseteq U$. For every subset A of X, $cl(A) \subseteq \delta cl(A)$ and so we have $cl(A) \subseteq U$ and hence A is rg-closed.
Counter Example 3.9 Let X={a,b,c}, τ={ϕ,X,{a},{a,b},{a,c}} Then the subset {a} is rg-closed but not $\delta(\delta g)^*$ -closed in (X, τ).

Proposition 3.10 Every $\delta(\delta g)^*$-closed set is gpr-closed but not conversely.

Proof: Let A be $\delta(\delta g)^*$-closed set and U be any regular open set containing A in X. By remark 2.3.every regular open set is δg-open and A is $\delta(\delta g)^*$-closed, $\delta cl(A) \subseteq U$. For every subset A of X, $pcl(A) \subseteq \delta cl(A)$ and so we have $pcl(A) \subseteq U$ and hence A is gpr- closed.
Counter Example 3.11 Let X={a,b,c}, τ={ϕ,X,{a}} Then the subset {b} is gpr-closed but not $\delta(\delta g)^*$ -closed in (X, τ).

Proposition 3.12 Every $\delta(\delta g)^*$-closed set is $\delta g^\#$ -closed but not conversely.

Proof: Let A be $\delta(\delta g)^*$ -closed and U be any δ-open set containing A in X. By remark 2.3. every δ-open set is δg-open and A is $\delta(\delta g)^*$-closed, $\delta cl(A) \subseteq U$. Hence A is $\delta g^\#$ -closed.
Counter Example 3.13 Let X={a,b,c}, τ={ϕ,X,{a}}.Then the subset {a} is $\delta g^\#$ -closed but not $\delta(\delta g)^*$-closed in (X, τ).

Proposition 3.14 Every $\delta(\delta g)^*$-closed set is rwg-closed but not conversely.

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Proof : Let A be $\delta(\delta g)^{*}$-closed and U be any regular open set containing A in X. By remark 2.3.every regular open set is δg-open and A is $\delta(\delta g)^{*}$-closed, $\delta cl(A) \subseteq U$. As $int(A) \subseteq A$ we have $cl(int(A)) \subseteq cl(A) \subseteq \delta cl(A)$ and hence A is rwg-closed.

Counter Example 3.15 Let X={a,b,c}, τ={ϕ,X,{a,b}}. Then the subset {a} is rwg -closed, but not $\delta(\delta g)^{*}$- closed in (X, τ).

Proposition 3.16 Every $\delta(\delta g)^{*}$-closed set is gspr-closed but not conversely.

Proof: Let A be $\delta(\delta g)^{*}$-closed and U be any regular open set containing A in X. By remark 2.3.every regular open set is δg-open and A is $\delta(\delta g)^{*}$- closed, $\delta cl(A) \subseteq U$. By remark 2.4. $spcl(A) \subseteq \delta cl(A)$ and so we have $spcl(A) \subseteq U$ and hence A is gspr –closed.

Counter Example 3.17 Let X={a,b,c}, τ={ϕ,X,{a,b}}. Then the subset {b} is gspr-closed but not $\delta(\delta g)^{*}$- closed in (X, τ).

Proposition 3.18 Every $\delta(\delta g)^{*}$-closed set is πg-closed but not conversely.

Proof: Let A be $\delta(\delta g)^{*}$- closed and U be any π-open set containing A in X. By remark 2.3.every π-open set is δg-open and A is $\delta(\delta g)^{*}$-closed, $\delta cl(A) \subseteq U$. For every subset A of X, $cl(A) \subseteq \delta cl(A)$ and so we have $cl(A) \subseteq U$ and hence A is πg-closed.

Counter Example 3.19 Let X={a,b,c}, τ={ϕ,X,{a},{b,c}}. Then the subset {c} is πg-closed but not $\delta(\delta g)^{*}$- closed in (X, τ).

Proposition 3.20 Every $\delta(\delta g)^{*}$-closed set is πgp-closed but not conversely.

Proof: Let A be $\delta(\delta g)^{*}$-closed and U be any π-open set containing A in X. By remark 2.3.every π-open set is δg- open and A is $\delta(\delta g)^{*}$-closed, $\delta cl(A) \subseteq U$. By remark 2.4. $pcl(A) \subseteq \delta cl(A)$ and so we have $pcl(A) \subseteq U$ and hence A is πg-closed.

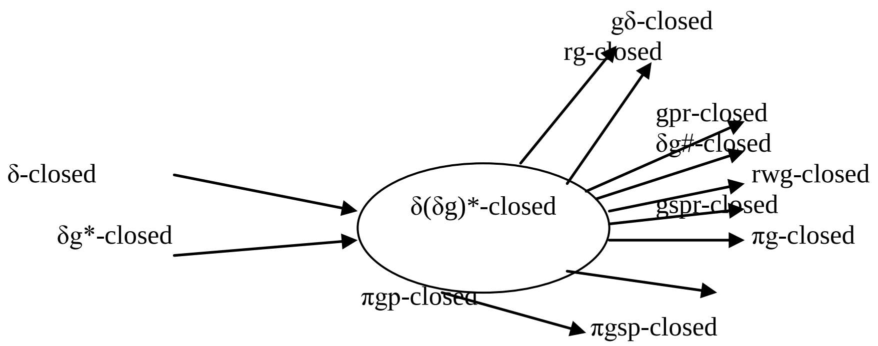
Counter Example 3.21 Let X={a,b,c}, τ={ϕ,X,{a}} Then the subset {a} is πgp-closed but not $\delta(\delta g)^{*}$-closed in (X, τ).

Proposition 3.22 Every $\delta(\delta g)^{*}$-closed set is πgsp-closed but not conversely.

Proof: Let A be $\delta(\delta g)^{*}$-closed set and U be any π-open set containing A in X. By remark 2.3.every π-open set is δg-open and A is $\delta(\delta g)^{*}$-closed, $\delta cl(A) \subseteq U$. By remark 2.4. $spcl(A) \subseteq \delta cl(A)$ and so we have $spcl(A) \subseteq U$ and hence A is πgsp-closed.

Counter Example 3.23 Let X={a,b,c}, τ={ϕ,X,{a},{b},{a,b}}. Then the subset {a} is πgsp-closed but not $\delta(\delta g)^{*}$-closed in (X, τ).

Remark 3.24 The following figure gives the dependence of $\delta(\delta g)^{*}$-closed set with eleven closed sets.



Remark 3.25 The following counter examples show that $\delta(\delta g)^{*}$-closedness is independent from g-closedness, δg-closedness, αg-closedness, *g-closedness, $\alpha\hat{g}$ -closedness, #gs-closedness, $\delta\hat{g}$ -closedness, gp-closedness and

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g*p-closedness.

Counter Example 3.26 Let X={a,b,c}, τ={ϕ,X,{a}}. In this topology the subset {b} is g-closed, δg-closed, αg-closed, *g-closed, $\alpha\hat{g}$ -closed, #gs-closed, $\delta\hat{g}$ -closed, g^*-closed, gp-closed and g*p-closed but not $\delta(\delta g)^*$-closed.

Counter Example 3.27 Let X={a,b,c}, τ={ϕ,X,{a},{a,b},{a,c}} In this topology the subset {a,b} is $\delta(\delta g)^*$-closed but not g-closed, δg-closed, αg-closed, *g-closed, $\alpha\hat{g}$ -closed, #gs-closed, $\delta\hat{g}$ -closed, gp-closed and g*p-closed.

Remark 3.28 The following Counter Example shows that $\delta(\delta g)^*$-closedness is independent from g*s-closedness.

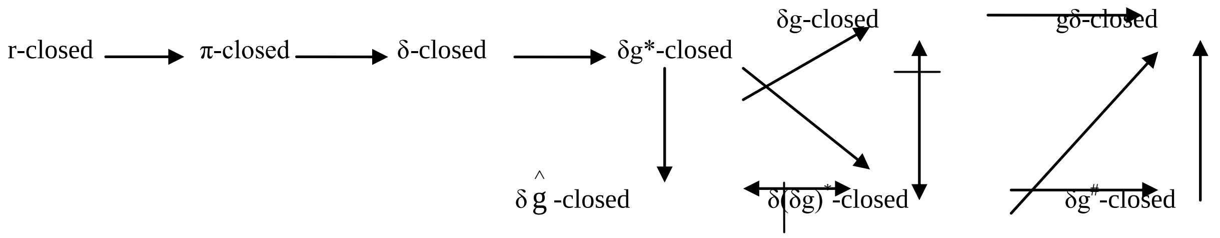
Counter Example 3.29 Let X={a,b,c}, τ={ϕ,X,{a},{a,b}}. In this topology the subset {a,c} is $\delta(\delta g)^*$-closed but not g*s-closed and the subset{b} is g^*s closed but not $\delta(\delta g)^*$-closed.

Remark 3.30 The following Counter Example shows that $\delta(\delta g)^*$-closedness is independent from (gs)*-closedness.

Counter Example 3.31 Let X={a,b,c}, τ={ϕ,X,{a,b}}. In this topology the subset {b,c} is $\delta(\delta g)^*$-closed but not (gs)*-closed.

Counter Example 3.32 Let X={a,b,c}, τ={ϕ,X,{a},{b,c}}. In this topology the subset {a,c} is (gs)*-closed but not $\delta(\delta g)^*$-closed.

Remark 3.33 From the above relations between various g-closed sets using δ-closure we get the following implications.



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