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# New Results on Vertex Prime Graphs

Dr. A. Selvam Avadayappan[1], R. Sinthu[2]

Associate Professor, Department of Mathematics, V.H.N.S.N. College, Virudhunagar, Tamil Nadu, India

Research Scholar, Department of Mathematics, V.H.N.S.N. College, Virudhunagar, Tamil Nadu, India

**ABSTRACT:**A graph G(V, E) is said to have a vertex prime labeling if its edges can be labeled with distinct integers from $\{1, 2, 3, \ldots, |E|\}$ such that for each vertex of degree at least 2, the greatest common divisor of the labels on its incident edges is 1. A graph that admits a vertex prime labeling is called a vertex prime graph.

In this paper, we prove that $mK_{3,3}$ and $mK_{4,4}$ are vertex prime graphs, where m is any positive integer.

**KEYWORDS**: labeling of graphs, vertex prime labeling of graphs.

**Subject Classification Code (2000):**05C(Primary)

## I.INTRODUCTION

Let G(V, E) be a graph. For notations and terminology, we follow [1]. G is called a vertex prime graph if there is a bijection $f : E \to \{1, 2, 3, \ldots, |E|\}$ such that for any vertex v, $\gcd_{uv \in E} \{f(uv)\} = 1$. The bijection f is called a vertex prime labeling of G.

For example, vertex prime labelings of some known graphs are illustrated in Figure 1.

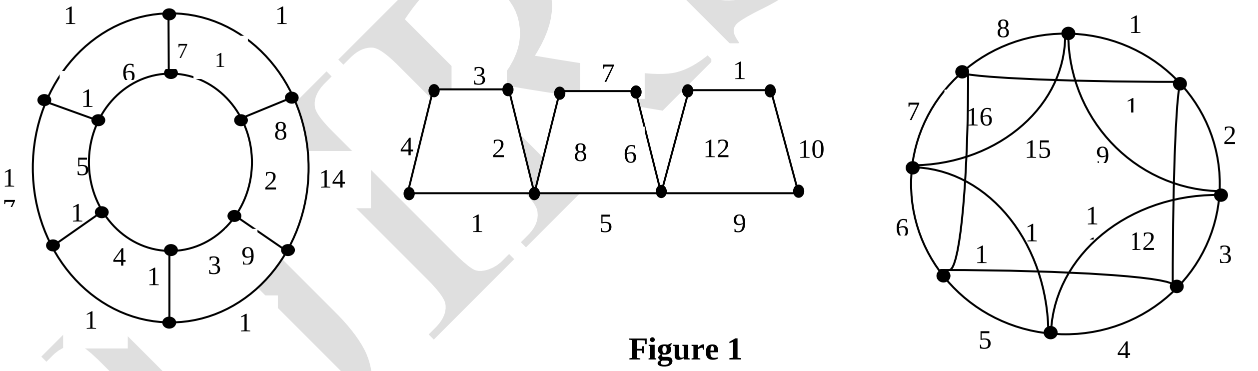

**Figure 1**

The concept of vertex prime graphs has been introduced by T. Deretsky, S.M. Lee and J. Mitchem [3] in 1991. They proved that the forests; any connected graph; $C_{2k} \cup C_n$; $C_{2k} \cup C_{2n} \cup C_{2k+1}$; $C_{2m} \cup C_{2n} \cup C_{2t} \cup C_k$; and $5C_{2m}$ are vertex prime. They have further proved that a graph with exactly 2 components, one of which is not an odd cycle has a vertex prime labeling and a 2 – regular graph with atleast two odd cycles does not have a vertex prime labeling. They have conjectured that a 2 – regular graph has a vertex prime labeling if and only if it does not have two odd cycles. Let $G = \bigcup_{i=1}^{t} C_{2n_i}$ and $N = \sum_{i=1}^{t} n_i$. In [2] I. Borosh, D. Hensley and A. Hobbs proved that there is a positive constant $n_0$ such that the conjecture of Deretsky et al., is true for the following cases:

i) G is the disjoint union of atmost seven cycles, or

ii) G is a union of cycles all of the same even length 2n if $n \leq 1,50,000$



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or if $n \geq n_0$, or

iii) $n_i \geq (\log N)^{4\log\log\log N}$ for all i = 1, 2, 3, . . . , t, or

iv) Each $C_{2n_i}$ is repeated atmostn$_i$ times.

In [4], SelvamAvadayappan and R. Sinthu proved that $mK_{2,3}$ is vertex prime.

In this paper, we prove the vertex primeness of the union of m disjoint copies of the complete graphs $K_{3,3}$ and $K_{4,4}$.

## II. BACKGROND OR RELATED WORK

Mean graphs and Super mean graphs are the related works.

## III. PRESENTATION OF THE MAIN CONTRIBTION OF THE PAPER / SCOPE OF RESEARCH

We prove that $mK_{3,3}$ and $mK_{4,4}$ are vertex prime graphs through the definition of vertex prime graphs.We also work on the general case of this theorem.

## IV. EXPERIMENTAL RESULTS

We proved that $mK_{3,3}$ and $mK_{4,4}$ are vertex prime graphs.

**Theorem 1** For any positive integer m, the graph $mK_{3,3}$ is a vertex prime graph.

**Proof**

Let $V(mK_{3,3}) = \{u_1^1, u_2^1, u_3^1; u_1^2, u_2^2, u_3^2; . . . ; u_1^m, u_2^m, u_3^m; v_1^1, v_2^1, v_3^1; v_1^2, v_2^2, v_3^2; . . . ; v_1^m, v_2^m, v_3^m\}$ and

$E(mK_{3,3}) = \{u_i^r v_j^r : 1 \leq i \leq 3, 1 \leq j \leq 3; 1 \leq r \leq m\}$

Define $f : E(mK_{3,3}) \rightarrow \{1, 2, 3, . . . , 9m\}$ as follows:

$f(u_1^r v_j^r) = 9(r - 1) + j, 1 \leq j \leq 3, 1 \leq r \leq m;$
$f(u_2^r v_j^r) = 9r - (j + 2), 1 \leq j \leq 3, 1 \leq r \leq m;$
$f(u_3^r v_j^r) = 9r - (j - 1), 1 \leq j \leq 3, 1 \leq r \leq m.$

Consider the vertex $u_1^r$. Clearly,

$\gcd((f(u_1^r v_j^r), 1 \leq j \leq 3, 1 \leq r \leq m))$
$= \gcd(9(r - 1) + j, 1 \leq j \leq 3, 1 \leq r \leq m)$
$= 1$

Similarly one can check for the remaining vertices $u_2^r$, $u_3^r$, $v_1^r$, $v_2^r$ and $v_3^r$.

Thus f is a vertex prime labeling of $mK_{3,3}$.
Hence $mK_{3,3}$ is a vertex prime graph.
For example, a vertex prime labeling of $8K_{3,3}$ is shown in Figure 2. ∎


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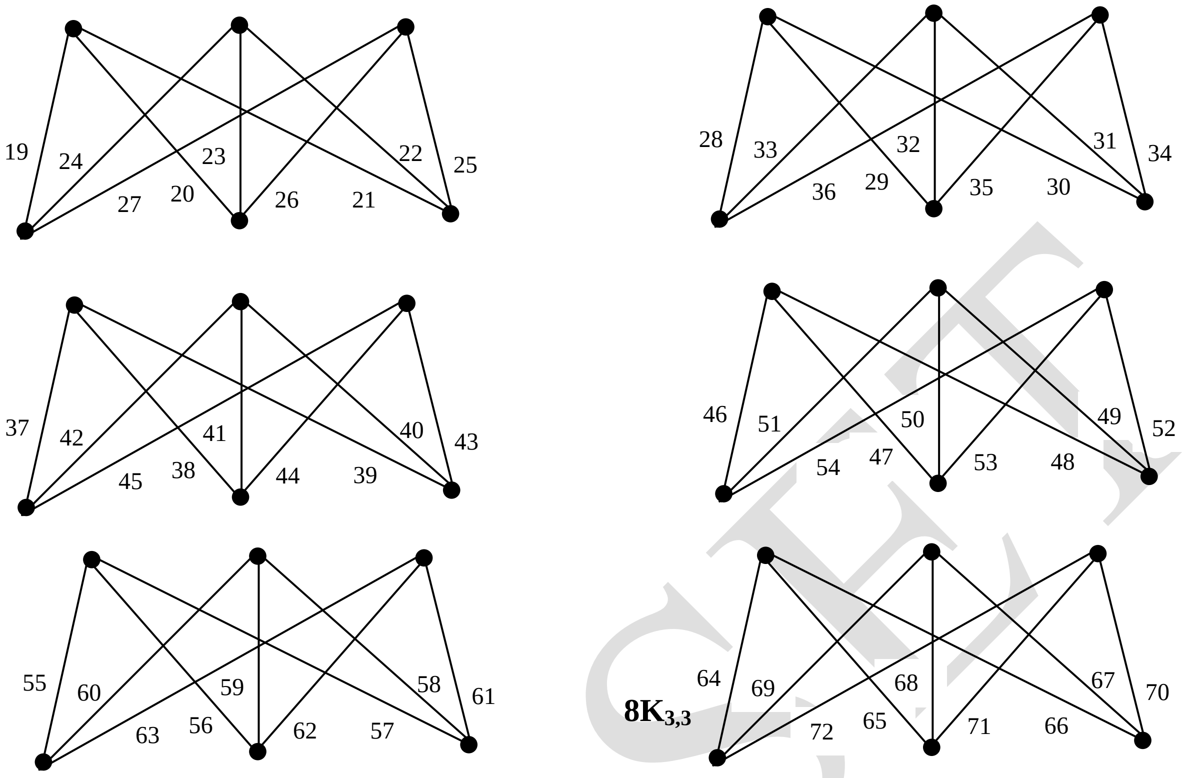


**Figure 2**

**Theorem 2** For any positive integer m, the graph $mK_{4,4}$ is a vertex prime graph.

**Proof**

Let $V(mK_{4,4}) = \{u_1^1, u_2^1, u_3^1, u_4^1; u_1^2, u_2^2, u_3^2, u_4^2; \ldots ; u_1^m, u_2^m, u_3^m, u_4^m; \quad v_1^1, v_2^1, v_3^1, v_4^1; v_1^2, v_2^2, v_3^2, v_4^2; \ldots ; v_1^m, v_2^m, v_3^m, v_4^m\}$

and

$E(mK_{4,4}) = \{u_i^r v_j^r : 1 \le i \le 4, 1 \le j \le 4; 1 \le r \le m\}$

Define $f : E(mK_{4,4}) \to \{1, 2, 3, \ldots, 16m\}$ as follows:

$f(u_1^r v_j^r) = 16(r - 1) + j, 1 \le j \le 4, 1 \le r \le m;$
$f(u_2^r v_j^r) = 16r - (j + 7), 1 \le j \le 4, 1 \le r \le m;$
$f(u_3^r v_j^r) = 16r - (j + 3), 1 \le j \le 4, 1 \le r \le m;$
$f(u_4^r v_j^r) = 16r - (j - 1), 1 \le j \le 4, 1 \le r \le m$

Consider the vertex $u_3^r$. Clearly,
$\gcd((f(u_3^r v_j^r), 1 \le j \le 4, 1 \le r \le m))$
$= \gcd(16r - (j + 3), 1 \le j \le 4, 1 \le r \le m)$
$= 1$

Similarly one can check for the remaining vertices $u_1^r$, $u_2^r$, $u_4^r$, $v_1^r$, $v_2^r$, $v_3^r$ and $v_4^r$.

Thus f is a vertex prime labeling of $mK_{4,4}$.
Hence $mK_{4,4}$ is a vertex prime graph.

For example, a vertex prime labeling of $14K_{4,4}$ is shown in Figure 3. ■

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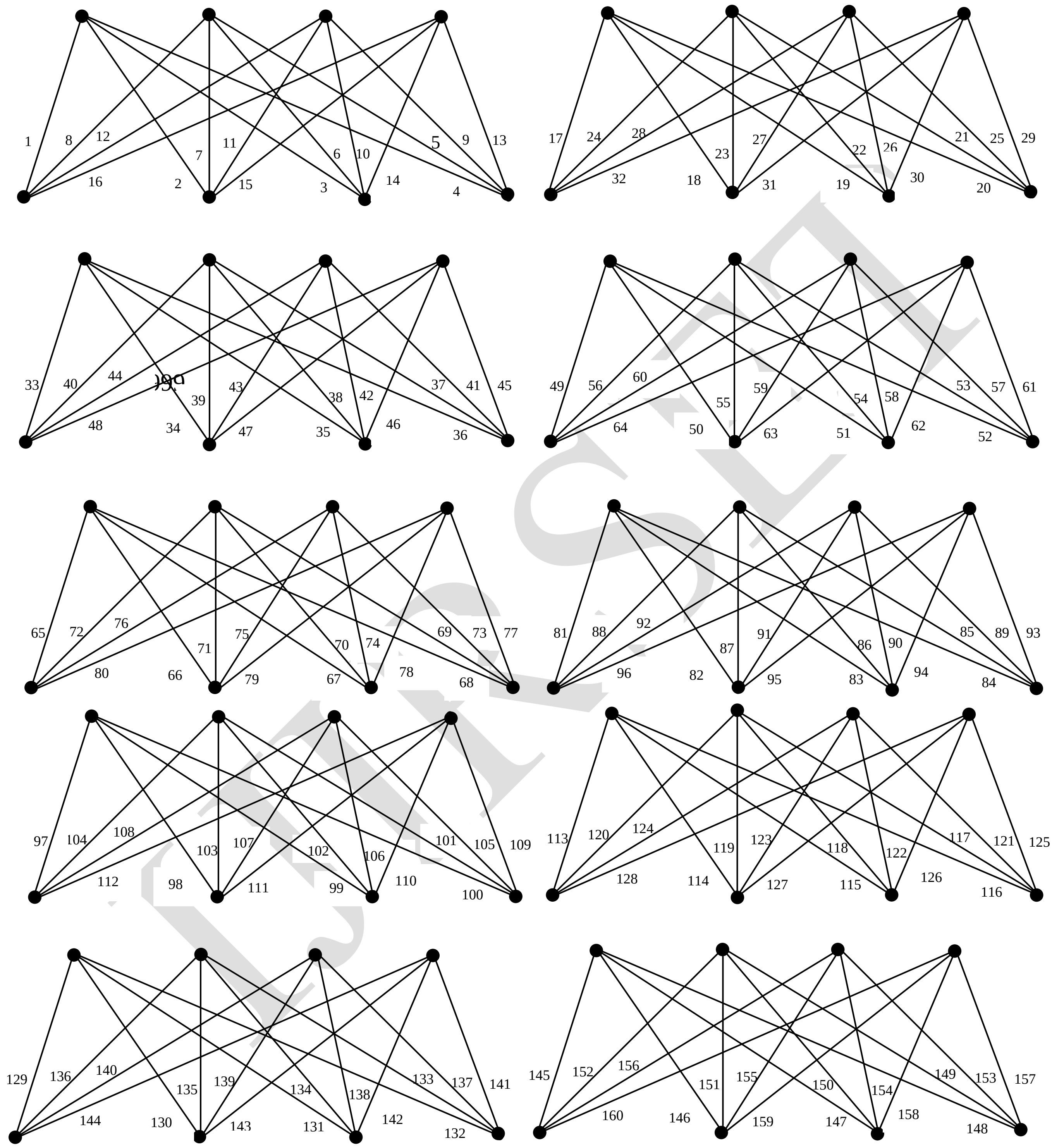

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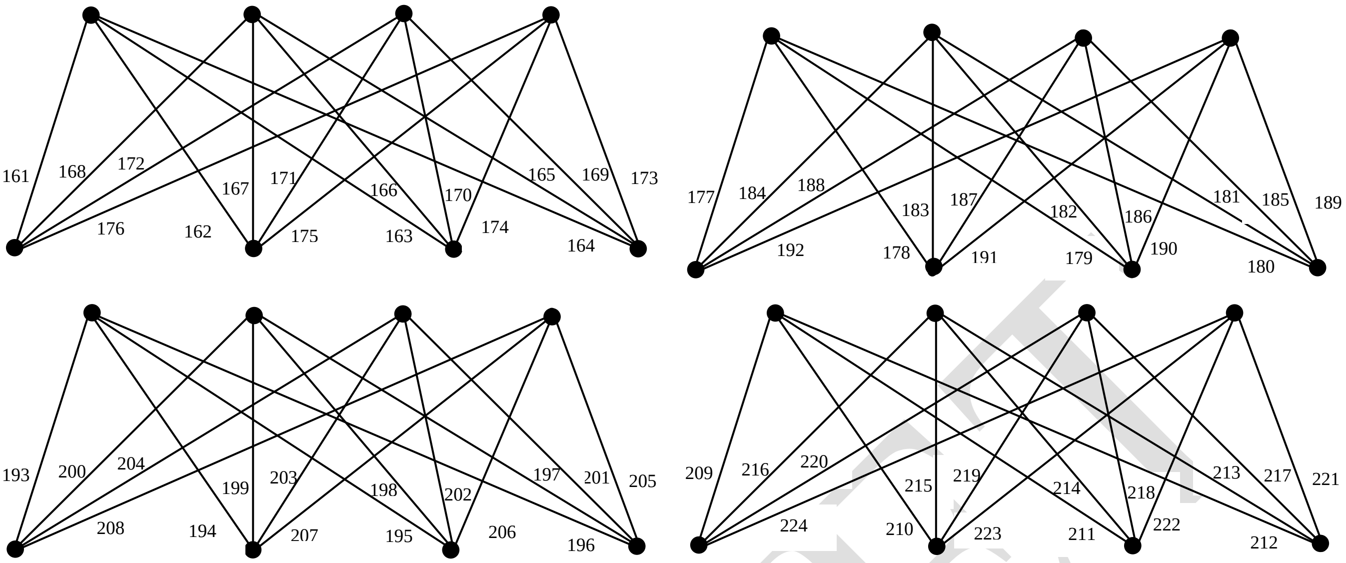


**$14K_{4,4}$**

**Figure 3**

## V.CONCLUSION

In this paper, we present Vertex prime labeling if its edges can be labeled with distinct integers. Thus we prove that $mK_{3,3}$ and $mK_{4,4}$ are vertex prime graphs. Some known graphs and unknown graphs are illustrated in a simple manner.

## VI.ACKNOWLEDGEMENT

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