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Certain Rodrigues Formula for F_1 and ϕ_1 Type Polynomials

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ABSTRACT

Suitable manipulations in Rodrigues type formulae can lead to numerous other type of formulae which are significatly important and are expected to find applications in probability theory and boundary value problems. Using Leibnitz formulae for n^{th} derivative of the product of two and three functions some elegant Rodrigues type formulae for polynomials corresponding to ϕ_1 function and Appell's F_1 function have been derived.

INTRODUCTION

Using Leibnitz formulae for n^{th} derivative of the product of two and three functions some elegant Rodrigues type formulae for polynomials corresponding to ϕ_1 function and Appell's F_1 function have been derived. In recent years, many authors have been consider the ϕ_1 function and Appell's F_1 type function in their studies [1-6]. In deriving the operational representations of various polynomials, we use the following fact that

$$D^{\mu}x^{\lambda}=\frac{\Gamma(1+\lambda)}{\Gamma(1+\lambda-\mu)}x^{\lambda-\mu}, D\equiv\frac{d}{dx} \tag{1}$$

Where λ and μ , $\lambda\geq\mu$ are arbitrary real numbers. In particular, use has been made of the following results [7-12]

$$D^{r}e^{-x}=(-1)^{r}e^{-x} \tag{2}$$

$$D^{r}x^{-\alpha}=(\alpha)_{r}(-1)^{r}x^{-\alpha-r} \tag{3}$$

$$D^{r}x^{-\alpha}=(\alpha)_{r}(-1)^{r}x^{-\alpha-r} \tag{4}$$

$$D^{r}x^{-\alpha-n}=(\alpha+n)_{r}(-1)^{r}x^{-\alpha-n-r} \tag{5}$$

$$D^{n-r}x^{\alpha-1+n}=\frac{(\alpha)_{n}}{(\alpha)_{r}}x^{\alpha-1+r} \tag{6}$$

$$D^{n-r}x^{-\alpha}=\frac{(\alpha)_n(-1)^n}{(1-\alpha-n)_r}x^{-\alpha-n+r},\alpha \text{ is not an integer} \tag{7}$$

where *n* and *r* are denote positive integers and

$$(\alpha)_n=\alpha\,(\alpha+1)\cdots\cdots(\alpha+n-1);(\alpha)_0=1 \tag{8}$$

Further,

$$D^n\left(uv\right)\equiv\sum_{r=0}^{n}{}^nC_rD^{n-r}uD^rv \tag{9}$$

$$D^n\left(uvw\right)\equiv\sum_{r=0}^{n}\sum_{s=0}^{n-r}{}^nC_r^{n-r}C_sD^{n-r-s}uD^rvD^sw \tag{10}$$

we also need the following definitions [4,5,13,14].

The Appell's F_1 function defined by

$$F_1[a,b,b';c;x,y]=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{m+n}(b)_m(b')_n}{(c)_{m+n}}\frac{x^m}{m!}\frac{x^n}{n!} \tag{11}$$

and the confluent hypergeometric function ϕ_1 is defined by

$$\phi_1[\alpha,\beta;\gamma;x,y]=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_{m+n}}\frac{x^m}{m!}\frac{x^n}{n!} \tag{12}$$

Rodrigues Type Formula

In this section we developed Rodrigues type formulae for F_1 and ϕ_1 functions

$$D^n\left\{x^{n+\alpha-1}(1-x)^{-\beta}(1+x)^{-\gamma}\right\}$$

$$=(\alpha)_nx^{\alpha-1}(1-x)^{-\beta}(1+x)^{-\gamma}F_1\left[\begin{matrix}-n;\beta,\gamma; & \frac{x}{1-x},\frac{x}{1+x}\\ \alpha; & \end{matrix}\right] \tag{13}$$

$$D^n\left\{x^{n+\alpha-1}(1-x^2)^{-\beta}\right\}$$

$$=(\alpha)_nx^{\alpha-1}(1-x^2)^{-\beta}F_1\left[\begin{matrix}-n;\beta,\beta; & \frac{x}{1-x},\frac{x}{1+x}\\ \alpha; & \end{matrix}\right] \tag{14}$$

$$D^n\left\{x^{-\alpha}(1-x)^{-\beta}(1+x)^{-\gamma}\right\}$$

$$=(\alpha)_nx^{-\alpha-n}(1-x)^{-\beta}(1+x)^{-\gamma}F_1\left[\begin{matrix}-n;\beta,\gamma; & \frac{x}{1-x},\frac{x}{1+x}\\ 1-\alpha-n; & \end{matrix}\right] \tag{15}$$

$$D^n\left\{x^{-\alpha}(1-x^2)^{-\beta}\right\}$$

$$=(\alpha)_nx^{\alpha-1}(1-x^2)^{-\beta}F_1\left[\begin{matrix}-n;\beta,\beta; & \frac{x}{1-x},\frac{x}{1+x}\\ 1-\alpha-n; & \end{matrix}\right] \tag{16}$$

$$D^n\left\{x^{\alpha-1+n}e^{-mx}(1-x)^{-\beta}\right\}$$

$$=(\alpha)_nx^{\alpha-1}(1-x)^{-\beta}e^{-mx}\phi_1\left[\begin{matrix}-n;\beta; & mx,\frac{x}{1-x}\\ \alpha; & \end{matrix}\right] \tag{17}$$

$$D^n\left\{x^{\alpha-1+n}e^{-mx}(1+x)^{-\beta}\right\}$$

$$=(\alpha)_nx^{\alpha-1}(1+x)^{-\beta}e^{-mx}\phi_1\left[\begin{matrix}-n;\beta; & mx,\frac{x}{1+x}\\ \alpha; & \end{matrix}\right] \tag{18}$$

$$D^n\left\{x^{\alpha-1+n}e^{-mx}x^{-\beta}\right\}$$

$$=(\alpha)_nx^{\alpha-\beta-1}e^{-mx}\phi_1\left[\begin{matrix}-n;\beta; & mx,1\\ \alpha; & \end{matrix}\right] \tag{19}$$

$$D^n\left\{x^{\alpha-1+n}e^{-mx}e^{-kx}\right\}$$

$$=(\alpha)_nx^{\alpha-1}e^{-mx}e^{-kx}\phi_1\left[\begin{matrix}-n;k; & x,x\\ \alpha; & \end{matrix}\right] \tag{20}$$

Proof of 13:

$$D^n\left\{x^{n+\alpha-1}(1-x)^{-\beta}(1+x)^{-\gamma}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n\ \ r} \frac{(-\ \ )_{r\ \ s}(-1)^{r\ \ s}}{r!s!} D^{n-r-s} x^{\alpha+n-}\ D^{s}(1-x)^{-\beta} D^{r}(1+x)^{-\gamma}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} \frac{(\alpha)_n}{(\alpha)_{r+s}} x^{\alpha+r+s-1}(-1)^{s}(\beta)_{s}(1-x)^{-\beta-s}(-1)^{r}(\gamma)_{r}(1+x)^{-\gamma-r}$$

$$= (\alpha)_n x^{\alpha-1}(1-x)^{-\beta}(1+x)^{-\gamma} F_1\left[\begin{matrix} -n;\beta,\gamma; \\ \alpha\,; \end{matrix} \qquad \frac{x}{1-x}, \frac{x}{1+x}\right]$$

Proof of 16:

$$D^{n}\left\{x^{-\alpha}(1-x^{2})^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} D^{n-r-s}\left\{x^{\alpha+n-1}\right\} D^{s}\left\{(1-x)^{-\beta}\right\} D^{r}\left\{(1+x)^{-\gamma}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!}$$

$$\times \frac{(\alpha)_n}{(1-\alpha-n)_{r+s}} x^{-\alpha+r+s-n}(-1)^{s}(\beta)_{s}(1-x)^{-\beta-s}(-1)^{r}(\gamma)_{r}(1+x)^{-\gamma-r}$$

$$= (\alpha)_n x^{\alpha-n}(1-x^{2})^{-\beta} F_1\left[\begin{matrix} -n;\beta,\beta; \\ 1-\alpha-n\,; \end{matrix} \qquad \frac{x}{1-x}, \frac{x}{1+x}\right]$$

Proof of 17:

$$D^{n}\left\{x^{\alpha-1+n}e^{-mx}(1-x)^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} D^{n-r-s}\left\{x^{\alpha+n-1}\right\} D^{s}\left\{e^{-mx}\right\} D^{r}\left\{(1-x)^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} \frac{(\alpha)_n}{(\alpha)_{r+s}} x^{\alpha+r+s-1}(-1)^{s}(m)^{s}e^{-mx}(-1)^{r}(\beta)_{r}(1-x)^{-\beta-r}$$

$$= (\alpha)_n x^{\alpha-1}(1-x)^{-\beta}e^{-mx}\phi_1\left[\begin{matrix} -n;\beta; \\ \alpha\,; \end{matrix} \qquad mx, \frac{x}{1-x}\right]$$

Proof of 18:

$$D^{n}\left\{x^{\alpha-1+n}e^{-mx}(1+x)^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} D^{n-r-s}\left\{x^{\alpha+n-1}\right\} D^{s}\left\{e^{-mx}\right\} D^{r}\left\{(1+x)^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} \frac{(\alpha)_n}{(\alpha)_{r+s}} x^{\alpha+r+s-1}(-1)^{s}(m)^{s}e^{-mx}(-1)^{r}(\beta)_{r}(1+x)^{-\beta-r}$$

$$= (\alpha)_n x^{\alpha-1}(1-x)^{-\beta}e^{-mx}\phi_1\left[\begin{matrix} -n;\beta; \\ \alpha\,; \end{matrix} \qquad mx, \frac{x}{1+x}\right]$$

Proof of 19:

$$D^{n}\left\{x^{\alpha-1+n}e^{-mx}x^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} D^{n-r-s}\left\{x^{\alpha+n-1}\right\} D^{s}\left\{e^{-mx}\right\} D^{r}\left\{x^{-\beta}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} \frac{(\alpha)_n}{(\alpha)_{r+s}} x^{\alpha+r+s-1}(-1)^{s}(m)^{s}e^{-mx}(-1)^{r}(\beta)_{r}x^{-\beta-r}$$

$$= (\alpha)_n x^{\alpha-\beta-1}e^{-mx}\phi_1\left[\begin{matrix} -n;\beta; \\ \alpha\,; \end{matrix} \qquad mx, 1\right]$$

Proof of 20:

$$D^{n}\left\{x^{\alpha-1+n}e^{-mx}e^{-kx}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} D^{n-r-s}\left\{x^{\alpha+n-1}\right\} D^{s}\left\{e^{-mx}\right\} D^{r}\left\{e^{-kx}\right\}$$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n-r} \frac{(-n)_{r+s}(-1)^{r+s}}{r!s!} \frac{(\alpha)_n}{(\alpha)_{r+s}} x^{\alpha+r+s-1}(-1)^s (m)^s e^{-mx}(-1)^r (k)_r e^{-kx}$$

$$= (\alpha)_n x^{\alpha-1} e^{-mx} e^{-kx} \phi_1 \begin{bmatrix} -n;k; & & x,x \\ \alpha\,; & & \end{bmatrix}$$

Similar way one can prove all the equations.

CONCLUSION

We conclude this paper by noting that, the results deduced above are significant and can lead to yield numerous other Rodrigues type formulae involving various special functions by suitable manipulations. More importantly, they are expected to find some applications in probability theory and boundary value problems.

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