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A Noval Solution of Implementation Issues of Kalman Filter for Tracking the Targets

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ABSTRACT: **The Kalman filter gives a linear, unbiased and minimum error variance recursive algorithm to optimally estimate the unknown state of a system from noisy data taken at discrete real-time. This paper present the practical aspects of implementing Kalman filter estimator applied to time varying stochastic non-linear models. The conventional implementation of the Kalman filter is particularly sensitive to round off errors, errors in the linearization process and ill conditioning in connection with matrix inversion. To reduce the complexity, the methods based on UD factorizations, aiming to simplify the update of the sate co-variance matrix $P=UDU^T$. During the update of P, there is no need to find the F_K, the Jacobian of the state transition matrix. Simulation of linear non linear target trajectory is performed using MATLAB R2009b. The result shows that the relative improvement in the convergence achieved using the UD factorization method.**

Keywords: **State vector, state transition function, covariance matrix, Jacobian matrix, filter gain**

I. INTRODUCTION

The Kalman Filter represents one of the most widely applied and demonstrably useful tool, to emerge from the state variable approach of “Modern control theory”[1]. The Kalman filter is a prediction filter. Put in simple terms, the filter extracts the noise free measurements from a set of erroneous measurements, by estimating the state of the plant, whose parameters are measured. In doing so it tries to minimize the difference between the measurements and an estimate of the measurements. In fact, the original proposition by Gauss regarding the estimation by the method of least squares becomes the Kalman Filter, when the minimization problem, as given by Gauss, is given a recursive solution [Gauss to Kalman]. It is often observed that for linear systems, the deterministic theory based on mean square estimation and the probabilistic estimation theory are equivalent.

II. THEORY OF KALMAN FILTER

As illustrated in Fig.1 consider a system represented by the dynamic equation

$$x(k+1) = f(x(k) + w(k) \quad (1)$$

where f() is a nonlinear function that explains the state transition of the a system represented by the sate x(k) (w(k) is the plant noise, assumed to be Gaussian). A measurement device gets a set of measurements regarding the plant given

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}(k)) + \boldsymbol{\nu}(k), \quad (2)$$

where **h ()** is a non linear function that transforms the state into the measurements. In other words, the measurements can be spelt out in terms of the state. ν (k) is the measurement noise, which is also normally distributed. Given the measurement x(k) at time k, the filter updates the state **x**(k) in terms of the difference between the current measurement and an estimate of the measurement made on the basis of the past (k-1) estimates.

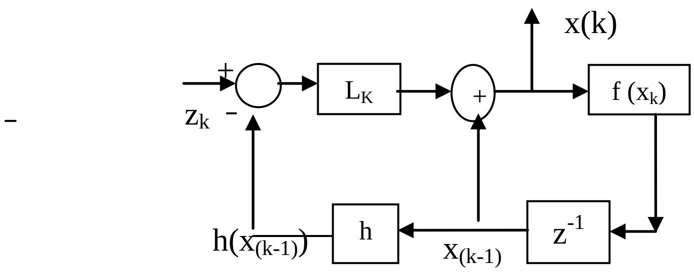

Fig.1 Block diagram of computational scheme

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$\mathbf{x}(k) = \mathbf{x}(k-1) + L(k)[\,\mathbf{z}(k) - \mathbf{h}(\mathbf{x}(k-1))]$ (3)

L(k) is the Kalman gain, computed from the co-variance of the state vector and the Jacobean of the vector functions **f** and **h**. The updated state undergoes a temporal update using Eq. 1 generating x(k+1), which become the candidate for estimating the measurement at time k+1. As the filter converges, the variance of residual z(k) – h(x(k-1)) , remains bounded to specified limits.

III. EXTENDED KALMAN FILTER

A. *Introduction*

Well-known method for estimation of state in linear systems with correlated noise is the extended Kalman filter, where the unknown parameters are estimated as a part of an enlarged state vector. In Extended Kalman Filter to estimate the state, by modelling the systems by state equations where the state consists of the system and noise parameters while the corresponding output, input and computed residuals are collected in the observation matrix of the state equations. The Kalman Filter requires an initial state for each object, and that initial state estimate must be obtained by detecting it. The system state vector x_k is assumed to be describing a dynamic system having the form,

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + w_k$$

and the measurements are described in a form

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + v_k$$

where w_k represents a white noise sequence v_k represent the measurement errors that occur at each observation time. Suppose m measurement quantities are available at discrete instants of time and are denoted at each time t_k as $\boldsymbol{z}_k$. Due to the measurement noise v_k there is a difference between the observed value and estimated value and is called the residual. The residual associated with the k^{th} measurement is $\mathbf{r}_k = \boldsymbol{z}_k$-h ($\mathbf{x}^{\wedge}$k/k-1) where h ($\boldsymbol{x}$(k/k-1)) is the estimate of the state by using previous measurement. The estimate is given as the linear combination of estimate predicted in the absent of new data and the residual $\mathbf{r}_k$. Thus, the mean square estimate is $\boldsymbol{x}^{\wedge}(k/k) = \boldsymbol{x}^{\wedge}(k/k-1) + L(k)\,[\,\mathbf{z}_k - h(\boldsymbol{x}^{\wedge}k/k-1)]$ where L (k) is the Kalman filter gain. State updating or to obtain an approximate state at t_{k+1} is $\boldsymbol{x}(k+1,k) = \boldsymbol{f}(\boldsymbol{x}^{\wedge}(k/k))$ where, $\boldsymbol{f}$ is the state transition function. Fig.1 illustrates the structure of Kalman filter. As the filter converges, the state sequence will represent the plant behaviour and the estimated measurement shall be free from noise. Also, the mean value of residual would to zero.

B. *Discrete kalman Filter Structure*

The Kalman filter provides an estimate of the state of the system at the current time based on all measurements of the system obtained up to and including the present time.

The system that is considered is composed of two equations

1. State equation:

$$x(k+1) = f(x(k),k) + G(k)w(k)$$

Where x (k) = n-dim state vector at time k,f($\boldsymbol{x}$ (k), k) = n-dim continuously differentiable vector function

$\boldsymbol{G}$ (k) = n*p random noise matrix, w(k) = p-dim state disturbance

2. Observation (measurement) model:

Suppose that m measurement quantities are available at discrete instants of time and are denoted at each time t_k as z_k .

$$z(k) = h(x(k),k) + v(k)$$

where, z = m-dim measurement vector, $\boldsymbol{h}$ ($\boldsymbol{x}$(k),k] = m-dim continuously differentiable vector function

v(k) = m-dim measurement noise process

$\{w_k\}$and$\{v_k\}$represent independent white-noise sequences. The initial state x_0 has a mean value $x^{\wedge}_{0/-1}$ and covariance matrix $P_{0/-1}$ and is independent of the plant and measurement noise sequence. The noise sequences have zero mean and second-order statistics described by

$E[v_k v_j^T] = R_k\partial_{kj}$, $E[w_k w_j^T] = Q_k\partial_{kj}$ $E[v_k w_j^T] = 0$ for all k, j. Also $E[v_k x^T(0)] = 0$

$E[w_k x^T(0)] = 0$ for $k \geq 0$

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Expand nonlinear functions f (**x**(k-1), k-1), and h (**x** (k), k) in a Taylor series about the conditional means $x^{\wedge}$ (k -1/ k -1) and $x^{\wedge}$ (k / k -1) as

$$F(k-1)=\frac{\partial f[x(k\text{-}1,k-1)]}{\partial x(k-1)}\Big/ x(k-1)=x^{\wedge}(k-1,k-1)$$

Similarly,

$$H(k-1)=\frac{\partial h[x(k),k)]}{\partial x(k)}\Big/ x(k)=x^{\wedge}(k/k-1)$$

approximate dynamical models become

$$x(k)=F(k-1)x(k-1)+G(k-1)w(k-1)+U(k-1)$$
$$z(k)=H(k)x(k)+v(k)+Y(k)$$

Where *U* (*k*-1) and *Y* (*k*) are calculated on-line from equations

$$U(k-1)=f(x^{\wedge}(k-1/k-1,k-1)-F(k-1)x^{\wedge}(k-1/k-1)$$
$$Y(k)=h[x^{\wedge}(k/k-1,k)-H(k)x^{\wedge}(k/k-1)$$

An estimate $\mathbf{x}^{\wedge}$(k/k) of the state **x**(k) is to be computed from the data z_0, z_1… zk so as to minimize the mean square error in the estimate. An estimate $\mathbf{x}^{\wedge}$k/k is referred to as the filtered estimate of **x**(k) and is a linear combination of an estimation at t_{k-1} and the measurement data z(k).

$$\hat{x}(k/k)=\hat{x}(k/k-1,k)+L(k)[z(k)-z(k/k-1) \qquad (4)$$

where $\mathbf{x}^{\wedge}$(k/k-1) is an estimate at t_k by using t_{k-1} measurements

$$x^{\wedge}(k/k-1)=F(k-1)x^{\wedge}(k/k-1)+U(k-1)$$
$$= f[\mathbf{x}^{\wedge}(k\text{-}1/k\text{-}1), k\text{-}1]$$

L(k) is the Kalman filter gain

$L(k)=P(k/k-1)H^T(k)[H(k)P(K/k-1)H^T(k)+R(K)]^{-1}$ Where P(k/k-1) is covariance of the error in the predicted estimate and given by

$P(k/k-1)=F(k-1)P(k-1/k-1)F^T(k-1)+G(K-1)Q(k-1)G^T(k-1)$ P(k/k)is the updated state covariance of the error in the estimate $\mathbf{x}^{\wedge}$(k/k) given by

$$P(k/k)=P(k/k-1)-L(k)H(k)P(k/k-1)$$

Q(k) is the plant noise covariance matrix

R_k is the measurement noise covariance matrix

Initialization is provided by

P(0/-1) = **P**0

$\mathbf{x}^{\wedge}$(0/-1) = **x**0

$z^{\wedge}$(k/k-1) is predicted measurement using updated state

$x^{\wedge}$ (k/k-1)

$z^{\wedge}$(k/k-1) = H(k) $\mathbf{x}^{\wedge}$(k/k-1) + **h**[$\mathbf{x}^{\wedge}$(k/k-1),k] - H(k) $x^{\wedge}$(k/k-1)

= h[**x**(k/k-1),k]

Equation (4) becomes

$$\hat{x}(k/k)=\hat{x}(k/k-1,k)+L(k)[z(k)-h[x^{\wedge}(k/k-1),k]$$

State updating OR to obtain an approximate state at t_{k+1} is

x(k+1,k) = f ($x^{\wedge}$(k/k)) where, f is the transition function

At k^{th} step, following parameters are computed for estimation:

Assume initial conditions: $E(x_0) = x^{\wedge}_0$ and $E(x_0 x_0^T) = P_0$

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1. Compute $P(k)$ using $P(k\text{-}1)$, $F(k\text{-}1)$ and $Q(k\text{-}1)$
2. Compute $H(k)$ using updated state. $x\hat{}(k/k\text{-}1)$
3. Compute $L(k)$ using $P(k)$ (computed in step 1), $H(k)$ and $R(k)$
4. Compute $P(k/k)$ using $L(k)$ (computed in step 3) and $P(k)$ (computed in step 1)
5. Compute successive values of state recursively, i.e. $x\hat{}(k/k)$, using the computed values of $L(k)$, the previous estimated state and the input data $z(k)$
6. Update the state $\boldsymbol{x}(k+1/k)$, using $F(k)$ and $\boldsymbol{x}\hat{}(k/k)$

C Bierman UD Factorization

The system that is considered is composed of two equations (1) & (2).

By using the modified Cholesky decomposition technique, P is factored in the form $P = UDU^T$ where D is a diagonal matrix and U is an upper triangular matrix. Initial value U is a identity matrix and D is P0 itself.Consider a single scalar observation

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + v_k$$

Let n is the dimension of the state vector. At the k^{th} step,

1. Compute $V(k) = U(k)^T H(k)^T$ Where V is an n-vector, and n is the dimension of the state vector.
2. Compute $\sigma(k) = [Rk + V(k)^T D(k)V(k)]$ (a scalar)
3. Compute Kalman gain $L(k) = U(k)D(k)V(k)\sigma)\sigma - 1)^{-1}$
4. Compute $B(k)$ using V(k),D(k) and the measurement noise variance σ

$$B(k) = D(k)V(k)V(k)^T \sigma(k-1)^{-1}$$

4. Update the U and D matrix by using B(k) computed in step 3.

$$U(k+1) = U(k)B(k) \qquad D(k+1) = D(k)\,\sigma(k-1)\sigma)\sigma(^{-1}$$

5. Compute successive values of state recursively, i.e. $\mathbf{x}\hat{}(k/k)$, using the computed values of L (k), the previous estimated state and the input data z (k)

$$\hat{x}(k/k) = \hat{x}(k/k-1,k) + L(k)[z(k) - h[x^{\wedge}(k/k-1),k]$$

6. State updating OR to obtain an approximate state at t_{k+1} is $\mathbf{x}(k+1,k) = \mathbf{f}(\mathbf{x}^{\wedge}(k/k))$ where, f is the transition function

In UD Factorization method, the covariance matrix P is updated by updating U and D factors. These forms of the covariance update take advantage of diagonal and symmetric matrix forms to make the implementation more efficient

D. Design of Kalman Filter Illustration

1) For Linear Path: Let us consider tracking of a target in a xy plane and going in a straight line.. To develop techniques for estimating state variables x_1 (position along x axis at time k+1), x_2 (velocity along x axis at time k+1), y_1 (position along y axis at time (k+1) y_2 (velocity along x axis at time (k+1) based up on measurements at time k+1 (rang and azimuth angle) when target dynamics are liner and measurement equations are nonlinear. The nonlinear measurement equations make this estimation problem a nonlinear filtering problem in which best state estimate is conditional mean $x(k+1/k+1)$.

Now we can define a state vector x that consists of position and velocity.

$$x(k) = \begin{bmatrix} p_k \\ v_k \end{bmatrix}$$

T = Track update time

$w(k)$ = 2-dim zero mean white Gaussian process with covariance

$E[w(k)w^T(k)] = Q(k)\,\delta(k\text{-}j)$

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$Q(k) = \sigma^2$

Let assume $\sigma^2 = 4$

w(k) represents uncertainty in the target acceleration due to atmospheric condition.

Radar measurements of range and azimuth angle are given by

$$z(k) = \begin{bmatrix} r(k) \\ \theta(k) \end{bmatrix} = \begin{bmatrix} \left(x^2(k) + y^2(k)\right)^{1/2} \\ \tan^{-1}(y(k)/x(k)) \end{bmatrix} + v(k)$$

v (k) is a two dimensional zero–mean white Gaussian process with covariance, $R(k) = \begin{bmatrix} 10m^2 & 0 \\ 0 & 10^{-2}\,rad \end{bmatrix}$

Initial state is assumed to be an n-dimensional Gaussian random vector with zero-mean and covariance

$$p(0) = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 2000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 2000 \end{bmatrix}$$

The platform trajectory is described by the following equations

$x(k+1) = x_1(k) + T\ x_2(k)$ and $y(k+1) = y_1(k) + T\ y_2(k)$

where

$x_1(0) = 25{,}000m \quad x_2(0) = -400\ m/s$

$y_1(0) = 25{,}000m \quad y_2(0) = +400 m/s \quad T = 0.1\ s$

The Jacobian of the measurement equation is determined as follows:

$x(k) = [x_1(k) \quad x_2(k) \quad y_1(k) \quad y_2(k)\ ]^T$

$$H(k) = \frac{\partial h[x(k),\ k)]}{\partial x(k)} \Big/ x(k) = x\wedge(k/k-1) \quad \text{and} \quad F(k-1) = \frac{\partial f[x(k-1,k-1)]}{\partial x(k-1)} \Big/ x(k-1) = x\wedge(k-1,k-1)$$

Simulation of the extended Kalman filter for linear target tracking is shown in the accompanying Fig.(2). Fig. (2.a) shows the actual linear track of the object. Fig. (2.b) shows the track when the noise is added with the actual measurements.. Fig. (2.c) shows the output of the Kalman filter.. Fig (2.d) shows error (innovation) in the predicted states. Fig.(2.e) shows the output of the UD filter. UD factorization method generates the states correctly. Fig.(2.f) shows the residual variation in the estimated value.

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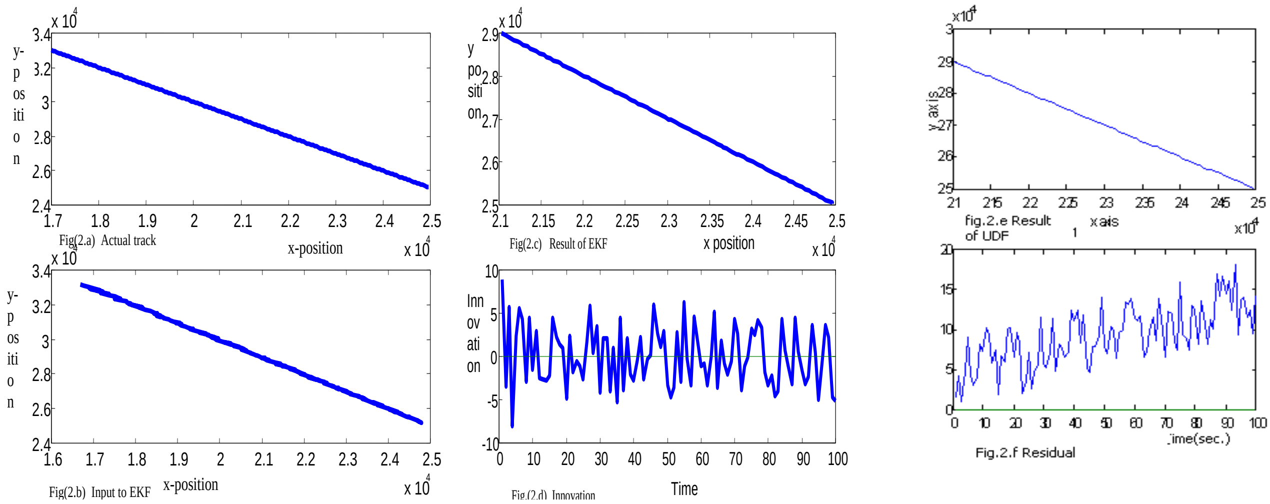

Fig(2.a) Actual track
Fig(2.b) Input to EKF
Fig(2.c) Result of EKF
Fig(2.d) Innovation
fig.2.e Result of UDF
Fig.2.f Residual

2) For non-Linear Path:

The Vander pol's equation is a second order system and described by the equation at critical condition ξ=1 is

$y_1(t) = (1 - y^2(t))y_2(t) + y_1(t) = 0$ and step size h = 0.09

Let $y_1(t)$ be denoted by $x_2(t)$ & $y_2(t)$ by $x_1(t)$. Then the equation becomes

$$x_1(k+1) = x_1(k) + h(1 - x_2^2(k))x_1(k) - hx_2(k)$$

$$x_2(k+1) = x_2(k) + hx_1(k)$$

Assume initial values

x_0 = [-2 1], and Step size h = 0.09;

Initial state is assumed to be an n-dimensional Gaussian random vector with zero-mean and covariance

$$P_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

The Jacobian of the measurement equation is determined as follows:

H_k =[0 1] where h2 = x_2 and $F(k-1) = \begin{bmatrix} 1 + h(1 - x_2^2(k)) & -h \\ h & 1 \end{bmatrix}$

Simulation result of the extended Kalman filter for this target trajectory are shown in the Fig.(3). Fig. (3.a) shows the actual trajectory of the object. Fig. (3.b) shows the track when the noise is added with the actual measurement data. This is the input of the Kalman filter. Fig.(3.c) shows the estimated trajectory. Fig.(3.d) shows the error(residual). Fig.(3.e) shows the output of the UD filter.. Fig.(3.f) shows the residual variation in the estimated value.

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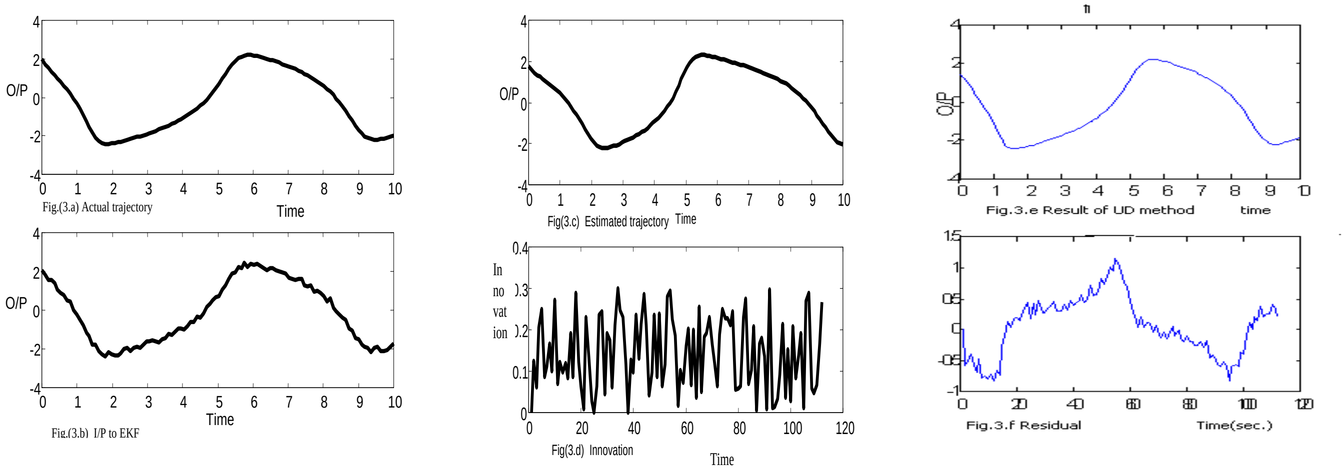

Fig.(3.a) Actual trajectory
Fig.(3.b) I/P to EKF
Fig(3.c) Estimated trajectory
Fig(3.d) Innovation
Fig.3.e Result of UD method
Fig.3.f Residual

IV. CONCLUSION

This work presents the practical aspects of implementing Kalman filter estimator applied to time varying stochastic non-linear models. The Kalman filter has found application in the tracking and navigation of all sorts of vehicles, and in predictive design of estimation and control systems. The UD Kalman filtering algorithm is considered efficient, stable and accurate for real time applications. In UD Factorization method, the covariance matrix P is updated by updating U and D factors. These forms of the covariance update take advantage of diagonal and symmetric matrix forms to make the implementation faster. This is one of the main advantages of the UD factorization method.

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