

An Accurate Subpixel Shift Registration in Noisy Image Using a Kernel Regression Method

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ABSTRACT: In this paper, a new accurate subpixel registration for pure shift estimation is proposed. The noise effect, which disturbs the quality of registration process, is taken into account. The kernel regression method which represents the field of nonparametric statistics is used as a tool for the estimation process due to its powerful capabilities in the field of both denoising and interpolation. The kernel regression depends on studying a local region intensities distribution and gradients. By applying gradient descent method, the global translation parameters can be estimated. Experimental results show that our proposed method can estimate the translation parameters accurately. Furthermore, our method performs well in noisy images.

KEYWORDS: Digital image processing, Pattern recognition, Nonlinear spatial filters, Subpixel registration, Kernel regression.

I. INTRODUCTION

Image processing often requires processing of multiple images of the same view captured from different positions with one or multiple cameras or maybe even with completely different types of sensors. Before this can be achieved, the pixels in the images must be somehow aligned to each other and this is achieved using registration. Image registration is the problem of finding a geometric transformation that maps the coordinate plane of one image to another using the image data itself.

What makes this problem harder is the fact that many applications demand quick, and precise registration. For some applications, it is critical to register input images with accuracy down to part of pixel level. Sub-pixel registration can also be hampered by the fact that the images may be contaminated by noise and aliasing, both of which may have a considerable effect, as well as possible lack of conformity of the image data to the proposed transformation model.

In this paper a novel subpixel registration method based on kernel regression [6] is proposed. The kernel regression is used not only as tool for interpolation but also for restoration and enhancement of noisy and possibly degraded images. Therefore, the kernel regression was adopted to estimate the pixels intensities. The steepest decent optimization is used in conjunction with the kernel regression to reach the required objective.

The paper is structured as follows. In section 2 a related work for subpixel registration is provided in addition to a background for the kernel regression for univariate data. In section 3 the proposed algorithm will be presented and explained. In section 4 the experiments and results will be shown. Finally the conclusion will be given in section 5.

II. RELATED WORK

For many image processing applications the image registration is considered important and highly needed. Many survey papers were provided for image registration methods [19,22]. The fields of applying image registration are varying in areas such as remote sensing [9], biological and medical imaging [8], and computer vision, with associated tasks ranging from superresolution [15], speckle and noise reduction [1], motion and change tracking, stereoscopic vision [14] and satellite image applications [4,12].

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Image registration methods may be classified in many ways but it has been suggested that nine-dimensional scheme would provide an excellent categorization [19].

Subpixel represents a special case of image registration since it is required in application where the high accuracy up to limits within the length of the pixel is required. A survey paper [16] described the most important methods for subpixel image registration. However, the most commonly used approach for subpixel registration is categorized based on interpolation [21]. Interpolation methods can be in form of correlation interpolation [16, 18], intensity interpolation [16], phase correlation interpolation [10, 16] and the geometric methods [3, 18]. It is obvious that the accuracy of these methods depends highly on the quality and efficiency of the interpolation algorithms [7].

The kernel regression as an effective tool for both denoising and interpolation in image processing was described in [6]. Its domain of applications are including fields of image reconstruction, and denoising. The kernel regression can be applied for both univariate and bivariate data. For the proposed algorithm in here, the bivariate version of the kernel regression was used.

The measurement model for image pixels is given by:

$$y_i = z(x_i) + \varepsilon_i, \quad i = 1, \dots, P \tag{1}$$

Where $z(\cdot)$ is the regression function, and ε_i 's are the independent and identically distributed zero mean noise values. In order to estimate the value of the function at any point x given the data, we can rely on a generic local expansion of the function about this point. Specifically, if x is near the sample at x_i we have the N -term Taylor series:

$$z(x_i) \approx z(x) + z'(x)(x_i - x) + \frac{1}{2!} z''(x)(x_i - x)^2 + \dots + \frac{1}{N!} z^{(N)}(x)(x_i - x)^N \tag{2}$$

$$= \beta_0 + \beta_1(x_i - x) + \beta_2(x_i - x)^2 + \dots + \beta_N(x_i - x)^N \tag{3}$$

The parameters $\{\beta_n\}_{n=1}^N$ will provide localized information on the n^{th} derivatives of the regression function. A least-squares formulation capturing this idea is to solve the following optimization problem:

$$\min_{\{\beta\}} \sum_{i=1}^P \left[y_i - \beta_0 - \beta_1(x_i - x) - \beta_2(x_i - x)^2 - \dots - \beta_N(x_i - x)^N \right]^2 \frac{1}{h} K\left(\frac{(x_i - x)}{h}\right) \tag{4}$$

Where h is a smoothing parameter (also called the “bandwidth”).

For the 2-D Case, it is similar to the 1-D case in (1), the data measurement model in 2-D can be modified from (1) where the coordinates of the measured data y_i is now the $(2 * 1)$ vector. Correspondingly, the local expansion of the regression function is given by:

$$\begin{aligned} z(x_i) &= z(x) + \{\nabla z(x)\}^T (x_i - x) + \frac{1}{2} (x_i - x)^T \{H z(x)\} (x_i - x) + \dots \\ &= z(x) + \{\nabla z(x)\}^T (x_i - x) + \frac{1}{2} \text{vec}^T \{H z(x)\} \text{vec} \{(x_i - x)(x_i - x)^T\} + \dots \end{aligned} \tag{5}$$

Where ∇ and H are the gradients $(2 * 1)$ and Hessian $(2 * 2)$ operators respectively. $\text{vec}(\cdot)$ is the vectorization operator which lexicographically orders a matrix into a vector. Defining $\text{vec}(\cdot)$ as the half-vectorization operator of the “lower-triangular” portion of a symmetric matrix. Example:

$$\text{vec} \begin{pmatrix} a & b \\ b & d \end{pmatrix} = [a \quad b \quad d]^T \quad \& \quad \text{vec} \begin{pmatrix} a & b & c \\ b & e & f \\ c & f & i \end{pmatrix} = [a \quad b \quad c \quad e \quad f \quad i]^T \tag{6}$$

Considering the symmetry of the Hessian matrix, equation (5) simplifies to:

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$$z(x_i) = \beta_0 + \beta_1^T (x_i - x) + \beta_2^T \text{vec}\{(x_i - x)(x_i - x)^T\} + \dots \tag{7}$$

A comparison of (5) and (7) suggests that $\beta_0 = z(x)$, is the pixel value of interest and the vectors β_1 and β_2 are:

$$\beta_1 = \nabla z(x) = \left[\frac{\partial z(x)}{\partial x_1}, \frac{\partial z(x)}{\partial x_2} \right]^T \tag{8}$$

$$\beta_2 = \left[\frac{\partial^2 z(x)}{\partial x_1^2}, 2 \frac{\partial^2 z(x)}{\partial x_1 \partial x_2}, \frac{\partial^2 z(x)}{\partial x_2^2} \right]^T \tag{9}$$

As in the case of univariate data, the β_n is computed from the following optimization problem:

$$\min_{\{\beta_n\}} \sum_{i=1}^p \left[y_i - \beta_0 - \beta_1^T (x_i - x) - \beta_2^T \text{vec}\{(x_i - x)(x_i - x)^T\} - \dots \right]^2 K_H(x_i - x) \tag{10}$$

With

$$K_H(x_i - x) = \frac{1}{\det(H)} K(H^{-1}t) \tag{11}$$

Where K is the 2-D realization of the kernel function, and H is the (2 * 2) smoothing matrix, It is also possible to express (10) in a matrix form as a weighted least-squares optimization problem [6]:

$$\begin{aligned} \hat{b} &= \arg \min_b \|y - X_x b\|^2 W_x \\ &= \arg \min_b (y - X_x b)^T W_x (y - X_x b) \end{aligned} \tag{12}$$

Given that:

$$\begin{aligned} y &= [y_1, y_2, \dots, y_p]^T \\ b &= [\beta_0, \beta_1^T, \dots, \beta_N^T]^T \end{aligned} \tag{13}$$

$$W_x = \text{diag}(K_H(x_1 - x), K_H(x_2 - x), \dots, K_H(x_p - x)) \tag{14}$$

With “diag” defining a diagonal matrix.

$$X_x = \begin{bmatrix} 1 & (x_1 - x)^T & \text{vec}^T\{(x_1 - x)(x_1 - x)^T\} & \dots \\ 1 & (x_2 - x)^T & \text{vec}^T\{(x_2 - x)(x_2 - x)^T\} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (x_p - x)^T & \text{vec}^T\{(x_p - x)(x_p - x)^T\} & \dots \end{bmatrix} \tag{15}$$

Regardless of the estimator order (N), since our primary interest is to compute an estimate of the image (pixel values), the necessary computations are limited to the ones that estimate the parameter. Therefore, the least-squares estimation is simplified to:

$$\hat{z} = \hat{\beta}_0 = e_1^T (X_x^T W_x X_x)^{-1} X_x^T W_x y \tag{16}$$

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Where e_1 is a column vector with the first element equal to one, and the rest equal to zero. Of course, there is a fundamental difference between computing β_0 for the $N = 0$ case, and using a high-order estimator $N > 0$ and then effectively discarding all estimated β_N except β_0 . Unlike the former case, the latter method computes estimates of pixel values assuming a N^{th} -order locally polynomial structure is present.

III. THE PROPOSED ALGORITHM

Subpixel image registration by iterative algorithm to a least squares error norm is a form of gradient-descent. It is an established method that has been implemented in multiple researches [2, 5, 13, 15, 17].

The proposed algorithm assumes that any regular algorithm for shift estimation method is applied to obtain the shift at integer-pixel level as a preprocessing step. To estimate the subpixel shift in an image, the gradient-descent optimization is used for minimizing the square differences in image intensities. The algorithm depends on the derivatives in both horizontal and vertical directions.

An image with subpixel shift in directions x_1 and x_2 is assumed to have shifts ∇_{x_1} and ∇_{x_2} , therefore to find the shifts the next minimization cost function should be solved:

$$F = \sum_{i=\text{all pixels}} (y_i^{ref} - y_i^{trans})^2 \tag{17}$$

Where F is the function to be minimized and represents the summation of square difference in intensities. y_i^{ref} is the reference image at pixel i , and y_i^{trans} is the translated image, y_i^{trans} is defined as:

$$y_i^{trans} = y_i(x_1 + \nabla_{x_1}, x_2 + \nabla_{x_2})$$

According to the gradient descent algorithm [11], the values of ∇_{x_1} and ∇_{x_2} are updated according to the following equations:

$$\nabla_{x_1}^{new} = \nabla_{x_1}^{old} + \alpha \frac{\partial F}{\partial x_1} \quad \& \quad \nabla_{x_2}^{new} = \nabla_{x_2}^{old} + \alpha \frac{\partial F}{\partial x_2} \tag{18}$$

Determining y_i^{trans} and the differential values $\partial F / \partial x_1$ and $\partial F / \partial x_2$ can be found through the following derivation:

$$\frac{\partial F}{\partial x_1} = \frac{\partial \left(\sum_{i=\text{all pixels}} (y_i^{ref} - y_i^{trans})^2 \right)}{\partial x_1} = 2 * \sum_{i=\text{all pixels}} (y_i^{ref} - y_i^{trans}) * \frac{\partial y_i^{trans}}{\partial x_1} \tag{19}$$

$$\frac{\partial F}{\partial x_2} = \frac{\partial \left(\sum_{i=\text{all pixels}} (y_i^{ref} - y_i^{trans})^2 \right)}{\partial x_2} = 2 * \sum_{i=\text{all pixels}} (y_i^{ref} - y_i^{trans}) * \frac{\partial y_i^{trans}}{\partial x_2} \tag{20}$$

Applying the kernel in equation (16) provides both estimated image due to translation y_i^{trans} in addition to the derivations in both directions $\frac{\partial y_i^{trans}}{\partial x_1}$ and $\frac{\partial y_i^{trans}}{\partial x_2}$. The updated ∇_{x_1} and ∇_{x_2} are used to calculate y_i^{trans} hence the

cost function in equation (17). The process is repeated till the cost change diverges to less than a given small value θ . A flow chart for the proposed algorithm is given briefly in Figure 1.

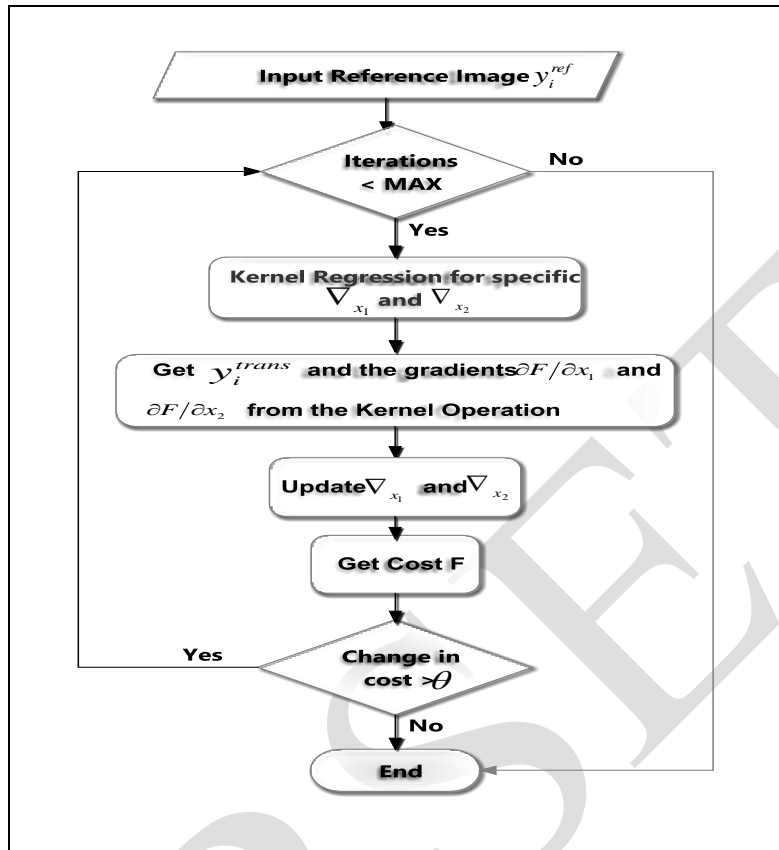


Fig. 1 A flow chart describes the proposed algorithm for obtaining the estimated shifts in both horizontal and vertical directions.

IV. EXPERIMENTS AND RESULTS

Two experiments were carried out to show the performance of the kernel regression. The first is to compare the accuracy results with well-known methods and the second it to show the impact of the bandwidth Factor h one the performance of the kernel method.

The first experiment: Four images, each of resolution 154 * 154 were taken as test images: Face, Cat, Bird, and beach as shown in Figure 2.

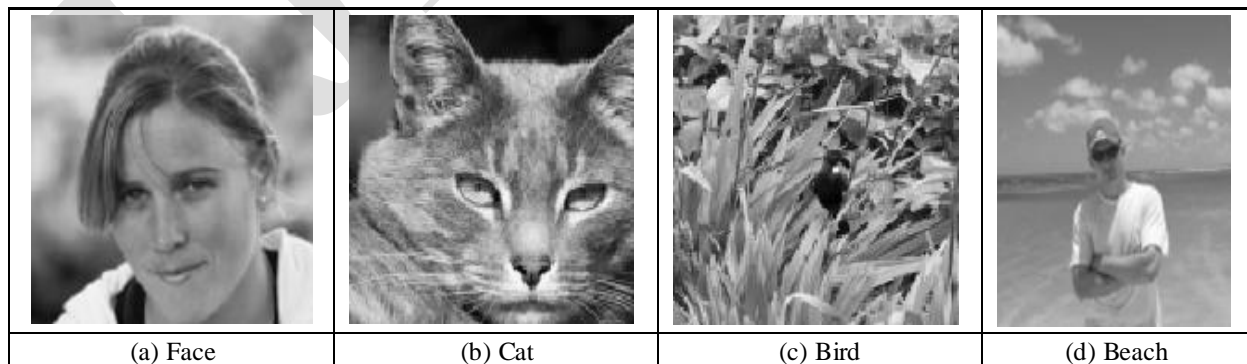


Fig. 2 Test image that will be used for examining the proposed algorithm

The images presented in Figure 2 were taken as reference images from [20]. The images were translated in both horizontal and vertical directions in specific subpixel values which are varying from 0.1 to 0.9. A linear interpolation method was used for generating the shifted images. Besides the shifting effect an additive white noise of zero mean and varying standard deviation was added to the shifted images. The measurements were taken also similar to in [20]. The subpixel registration error is determined as following:

$$\|e\|_2^2 = (\nabla_{x_1} - \hat{\nabla}_{x_1})^2 + (\nabla_{x_2} - \hat{\nabla}_{x_2})^2 \tag{21}$$

The equation above represents the RMS error for the estimated shift for single estimated shift in both directions .

To evaluate the proposed algorithm, a plot of the average of the root mean square registration errors for all values of the estimated shifts in both directions versus the noise strength (standard deviation) is plotted in Figure 3. Figure 3 plots not only the proposed algorithm results, but also compares between its performance against four methods for subpixel registration. Those methods were taken also as reference for evaluating subpixel registration in [20]. Those methods are Lucas-Kanade, the Fourier phase-based, Ncorr 2D quadratic, and global space-invariant resampling filter of size 4 * 4.

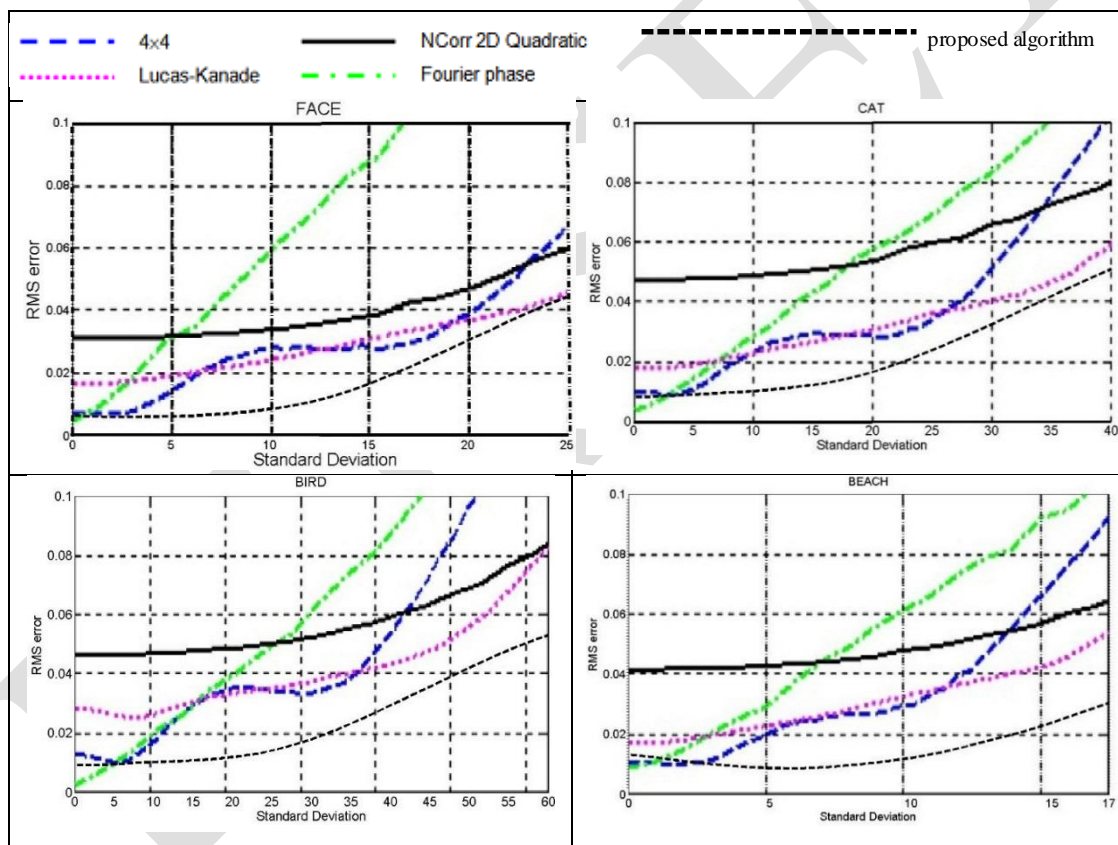


Fig. 3 The resulted RMS error against range of noise strengthes. The results are obtained after applying the proposed algorithm for the four test images

The second experiment: The bandwidth factor h will be tested as it is considered essential part of the kernel regression. The effect of the bandwidth factor can change the performance of the kernel since it controls the size of the area where the kernel is applied. One of the testing images, face, is subject to varying standard deviation as well as varying h 's. In Figure 4 the results for applying the kernel are shown.

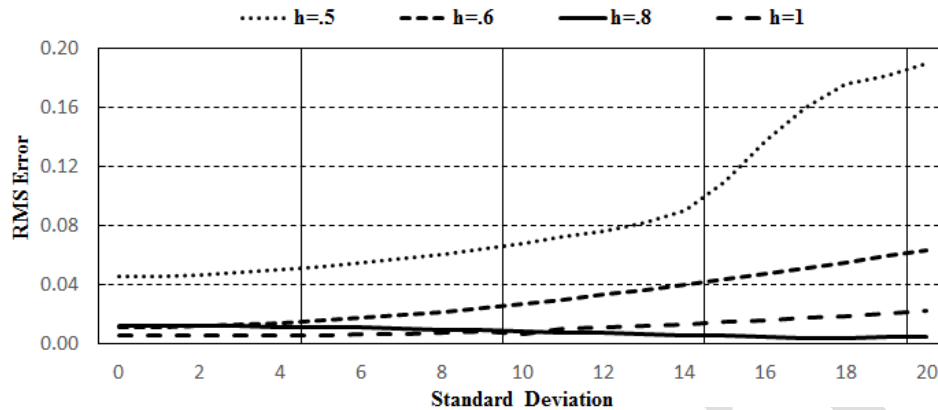


Fig 4: RMS Error at different Bandwidth h 's

Figure 4 shows the effect of different values of bandwidth $h=5, .6, .8, 1$. Those values were chosen to show how the proposed algorithm would be affected. The performance was taken also as RMS error resulted after applying the kernel. It is obvious that the bandwidth h affects the performance of the resulted accuracy and its performance varies at different values of standard deviation. For example, when $h = .8$ gives the best results at standard deviation of 11 while as $h = 1$ acts better in the first range of the standard deviation. That gives idea why is the motive for the use of the steering kernel regression

V. CONCLUSION AND FUTURE WORK

The subpixel registration is obvious an ill-posed problem where the translation model parameters is required. The solution for most of ill-posed problems can usually solved by optimization methods. To do so, we define a goal, also known as an objective (cost) function, for the inverse problem. The goal is a functional that measures how close the estimated data from the recovered model fits the observed data. The kernel regression has proved its superiority in the estimation for 2D images intensities and gradients. Another benefit for using the kernel is its ability to absorb the noise effect and the edge change since it is relying on the local structure of the pixels. The bandwidth factor has its own effect on the accuracy. It was set manually however it can be changed automatically if applying the steering kernel regression

As a future work to this paper, the author suggests applying steering kernel regression [6] in order to find the best bandwidth automatically.

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