ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

*(An ISO 3297: 2007 Certified Organization)*

**Vol. 3, Issue 6, June 2014**

# Cram´er- Rao Lower Bounds and Bayesian Cram´er- Rao Lower Bound Estimators for Blind Channel Estimation of SNR and Phase Noise

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**ABSTRACT:** Channel Estimation is the process of characterizing the effect of the physical medium on the input sequence and Orthogonal Frequency division multiplexing (OFDM) Systems are especially suited for channel estimation. In this work we have designed a Cram´er- Rao Lower Bound estimation for blind channel estimator of SNR in OFDM systems. Phase Noise is a time varying process which changes the symbol from time to time and has a high deteriorating effect on higher order OFDM system. We have derived a Bayesian Cram´er- Rao Lower Bound Estimator for estimating phase noise.

**KEY WORDS** : SNR, Phase Noise, Cram´er- Rao Lower Bound, Bayesian Cram´er- Rao Lower Bound

## I.INTRODUCTION

Multiple-input multiple-output (MIMO) technology can be utilized to enhance the throughput and reliability of wireless communication links by introducing multiplexing and diversity gains to the overall system [1], [2]. As a result, MIMO systems are an effective means to meet the stringent requirements on today's wireless communication systems that demand higher spectral efficiencies and throughputs [3]. On the other hand, phase noise severely deteriorates the performance of MIMO systems [4].Accurate SNR estimate is required for measuring the channel quality for adaptive modulation schemes as well as for soft decoding procedures as shown in [5], [6] and [7]. In addition to low-complexity requirement, it is essential to assess the truthfulness of SNR estimators in term of their statistical variances. For this purpose, the well-known CRLB is a prominent benchmark to evaluate the statistical variance performance of unbiased estimators.

Actually both data aided (DA) and non-data aided (NDA) trends are considered for either performance bounds derivation or estimation algorithms. Data aided approach, which relies on the transmission of known data streams such as training sequences and also pilot symbols, should expedite and ease the estimation process. Unfortunately, this approach limits the system through-put in the sense that adding known pilot symbols to the data stream should drop down the spectral efficiency of the communication system. Hence NDA SNR estimation approach receives substantial attention in recent literature. CRLB for NDA SNR estimation is derived in [8] from both BPSK and QPSK modulated signals with AWGN channel. Derived bounds are compared to those obtained for DA estimation. In [9], a straightforward approximation of the CRLB for NDA SNR estimation from BPSK modulated signals over AWGN channel is presented in efficient form that avoids tedious numerical integration. Authors, in [10], derive a lower bound for SNR estimation from general one/two dimensional modulation signals with axis/half plane symmetry over AWGN channel. Exact analytical CLRB of unbiased NDA SNR estimation from square QAM signals using I/Q received signal model is addressed in [11], where a generalization of the elegant CLRB expressions presented in [8] is also introduced.

Phase noise is a time varying process that changes from symbol to symbol [12]–[14]. Moreover, the deteriorating effect of phase noise may be more severe in MIMO systems employing higher order modulations, given that in MIMO systems, independent oscillators may be used at each transmit and receive antenna resulting in multiple phase noise processes that need to be jointly estimated at the MIMO receiver[13]. The use of independent oscillators at each transmit and each receive antenna is well motivated in applications where antennas need to be placed far apart from one

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International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

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another, e.g., in the case of line-of-sight (LoS) MIMO systems. As a result, even though Cram´er- Rao lower bounds (CRLBs) and algorithms for estimation of phase noise in single-input single-output (SISO) systems have been extensively and thoroughly studied in [12], these results cannot be applied to the case of MIMO systems. Similarly, phase locked loops, that can be used in SISO systems for phase noise tracking, cannot be applied in LoS-MIMO systems where multiple phase noise parameters need to be tracked simultaneously at the receiver [10].

The proposed work discusses a CRLB for NDA SNR estimation. Different types of time domain channel estimators are also studied. The performance of the proposed estimator is compared with the performance of ML estimator and Subspace blind channel estimator. Similarly the Bayesian Cram´er-Rao lower bounds (BCRLBs) for online, i.e., filtering, and offline, i.e., smoothing, estimation of phase noise over the length of a frame is also derived.

## II. OFDM SYSTEM FOR BLIND ESTIMATION

In OFDM systems, the serial data are converted into M parallel streams. Each parallel data stream modulates a different carrier. The frequency separation between the adjacent carriers is 1/T, where T is the symbol duration for the parallel data that is M times of the symbol duration for the serial data. Let us consider an OFDM signal in the interval (nT,(n+1)T) as [6]:

$$s(t) = \sum_{m=o}^{M-1} a_{m\,(n)} e^{jw_m t} \tag{1}$$

Where $a_m(n)$ are symbols resulting from a modulation constellation like 16 QAM. $w_m$ is the frequency of $m^{th}$ carrier that is $m\frac{2\pi}{T}$, and the M samples that are sampled at t = nT +( i* 1/T) with i = 0 ,1,2…,M-1 as follows:

$$s(nM+i) = \sum_{m=o}^{M-1} a_{m\,(n)} e^{j\frac{2\pi}{M}mi} \tag{2}$$

From this equation, the M samples can be seen as the inverse discrete Fourier transform (IDFT) of a block for M input symbols. Theoretically speaking, when the number of carriers is large enough, symbol duration *T* is much larger than the duration of FIR channel; IS1 is negligible. However, for the high-bit-rate communications, it is impractical to choose very large M to make ISI negligible. Therefore, a cyclic prefix of length P is added into each block of IDFT output at the transmitter. The length of the prefix is chosen to be longer than the length of the channel impulse response in order to avoid inter-block interference (IBI). That results with total cancellation of IS1 and inter carrier interference (ICI). The input data will be as follow:

$$s(n(M+P)+i) = \sum_{m=o}^{M-1} a_{m\,(n)} e^{j\frac{2\pi}{M}(i-P)} \tag{3}$$

Where *S (n (M+P) +i)* denotes the cyclic prefix.

## III. CRAM´ER- RAO LOWER BOUNDS AND BAYESIAN CRAM´ER- RAO LOWER BOUND ESTIMATORS

The model adopted in this case is very near to the classical system model that is consists of N sub-carriers and a cyclic prefix of length $N_g$. Supposing that $T_s$|denotes the sampling time and $v= N+ N_g$., the duration of OFDM symbol will be $T= vT_s$. In the case of a transmission over a multipath Rayleigh channel and for a nth transmitted OFDM symbol given as

$$x_{(n)} = [x_{(n)}\left[-\frac{N}{2}\right], x_{(n)}\left[-\frac{N}{2}+1\right] \dots\dots\dots\dots, x_{(n)}\left[\frac{N}{2}-1\right]]^T \tag{4}$$

Where ({ $x_{(n)}[b]$} are modulated 4-QAM or 16 QAM) and the nth received OFDM symbol

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ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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Vol. 3, Issue 6, June 2014

$$y_n = [\, y_n\left[-\frac{N}{2}\right], y_n\left[-\frac{N}{2}+1\right], \dots\dots\dots\dots\dots\dots\dots\dots y_{n\left[\frac{N}{2}-1\right]}\,]^T \quad (5)$$

$$y_{(n)} = H_{(n)} x_{(n)} + w_{(n)} \quad (6)$$

Where $W(n)$ N × 1 zero-mean complex Gaussian noise vector with covariance matrix $\sigma^2 I_N$ and $H_{(n)}$ is a N × N diagonal matrix with diagonal elements given by:

$$[H(n)]_{k.k} = \frac{1}{n}\sum_{l=1}^{L}\left[\alpha_l^{(n)}\ x\ e^{-j2\pi}\left(\frac{k-1}{N}-\frac{1}{2}\right)\pi l\right] \quad (7)$$

L is the total number of propagation paths; αl is the l th complex gain of variance with $\sum_l^L = 1\ \ \sigma_{\alpha l}^2$=1
The L individual elements are uncorrelated with respect to each other. They are wide sense stationary narrowband complex Gaussian processes, with the so-called Jakes' power spectrum with Doppler frequency $f_d$.. It means that $\alpha l^{(n)}$ are correlated complex Gaussian variables with zero -means and correlation coefficients given by:

$$R_{\alpha l}^{k} = E\left[\alpha_i^{(n)}\ \alpha_i^{(n-P)H}\right] = \sigma_{\alpha l}^2\ J_{0(2\pi f_d TK)} \quad (8)$$

Then, the observation model for the nth OFDM symbol can be re-written as

$$y_{(n)} = diag\{x_{(n)}\} F\alpha_{(n)} + \omega_{(n)} \quad (9)$$

Where $a_{(n)} = [\alpha_1^{(n)}, \dots\dots\dots\dots, \alpha_L^{(n)}]^T$ is L * 1 vector and F is the N * L Fourier matrix
In order to find benchmarks for the performance of phase noise estimators over the length of a frame, we have derived a Bayesian Cramer-Rao Lower Bounds (BCRLB) for offline and online estimation of time varying phase noise in an MIMO system. The BCRLB assumes no knowledge of the transmitted symbols, and can be derived by taking into account the a priori distribution of the transmitted symbols. Thus, to evaluate the BCRLB, the likelihood function $p(y|\emptyset)$ is achieved by averaging out $p(y|\emptyset, S)$ over a priori distribution of s. Accordingly, assuming all elements of *A* are equiprobable, the log likelihood function is given by

$$\log p(y(k)|\emptyset(k)) = \log \sum_{i=0}^{S-1} \frac{1}{S(\pi\,\sigma_\omega^2)^{N_r}}\ \ xexp\left\{-\frac{\|y(k)-\theta^{[r]}(k)H\theta^{[t]}(k)c_i\|^2}{\sigma^2\,\omega}\right\} \quad (10)$$

The estimates are calculated using both analytical and numerical methods. A 2 x 2 MIMO system used for estimating the phase noise variation.

## IV. RESULTS AND DISCUSSIONS

The LS estimator is shown to be the basic algorithm and gives regular results used with practically all the schemes of channel estimation, the LS estimator will be expressed as a ratio between the input data sequence and the output. The LMS estimator uses one tap LMS adaptive filter at each pilot frequency. The first value is found directly through LS and the following values are calculated based on the previous estimation and the current channel output. The LMS estimator is used mainly for the tracking of the channel and is usually clustered with an equalizer or a decision feedback equalizer. The LMMSE estimator is of considerable complexity, since a matrix inversion is needed every time

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ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
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Vol. 3, Issue 6, June 2014

the training data in exchanges. The Fig 1 gives the plot of different channel estimators typically used in OFDM systems.

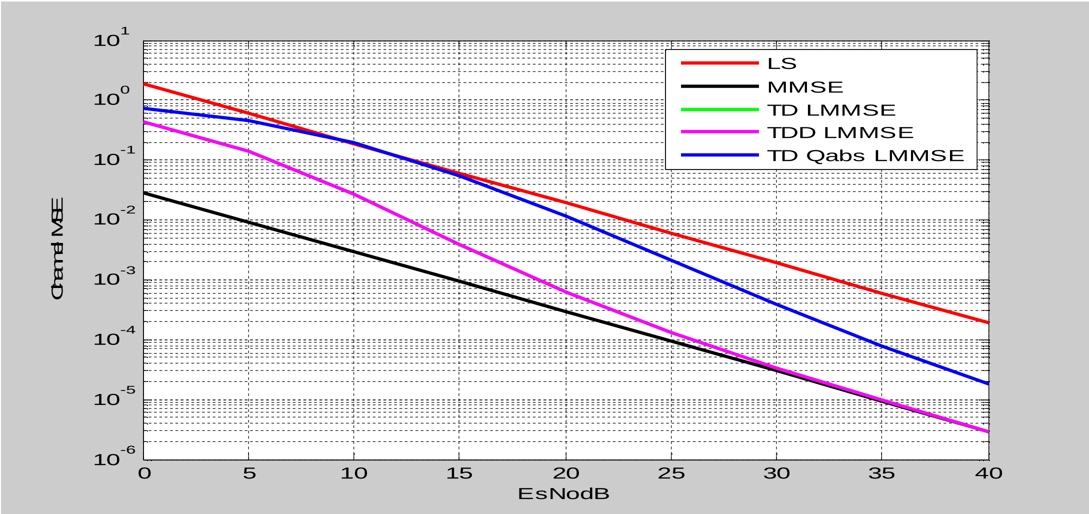


Fig. 1 Simulation of Different Channel Estimation Techniques

In Fig 1 the legend TD LMMSE refer to a time domain channel estimation technique, TDD LMMSE refer to the technique in which the channel covariance is ignored in the estimation and TD Qabs LMMSE refers to estimation in which the smoothing matrix is involved. LMMSE techniques are complex and computationally intensive TDD LMMSE and TD Qabs LMMSE reduces the complexity and computational time.

Maximum Likelihood estimation (MLE) is an important tool in determining the actual probabilities of the assumed model of communication. Maximum likelihood estimation is a method to determine these unknown parameters associated with the corresponding chosen models of the communication channel. The Cramer-Rao Lower Bound is widely used in statistical signal processing as a benchmark to evaluate unbiased estimators. In this work we have derived a CRLB for NDA estimation of SNR. Fig 2 gives the MSE performance of SNR for ML estimator and the designed CRLB estimator.

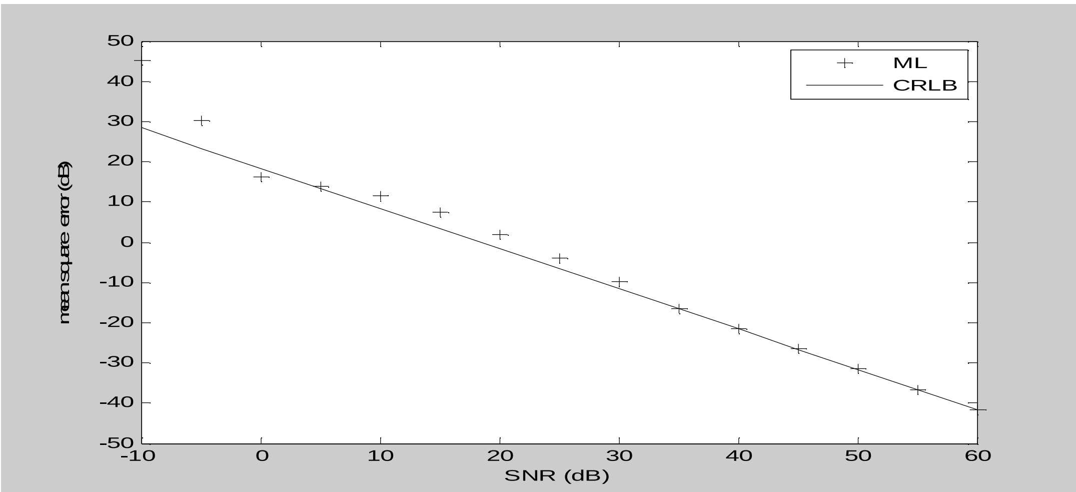


Fig. 2 MSE Performance versus SNR for ML and CRLB Estimators

ISSN (Print) : 2320 – 3765
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It can be observed from Fig 2 that the performance of both the estimators almost converge at high SNR but at low SNR the mse performance of CRLB is much better than ML estimator. This makes it possible the deployment of CRLB estimators for highly sensitive MIMO OFDM systems. Fig 3 gives the MSE performance of SNR estimation by subspace blind channel estimation and the proposed CRLB based estimator.

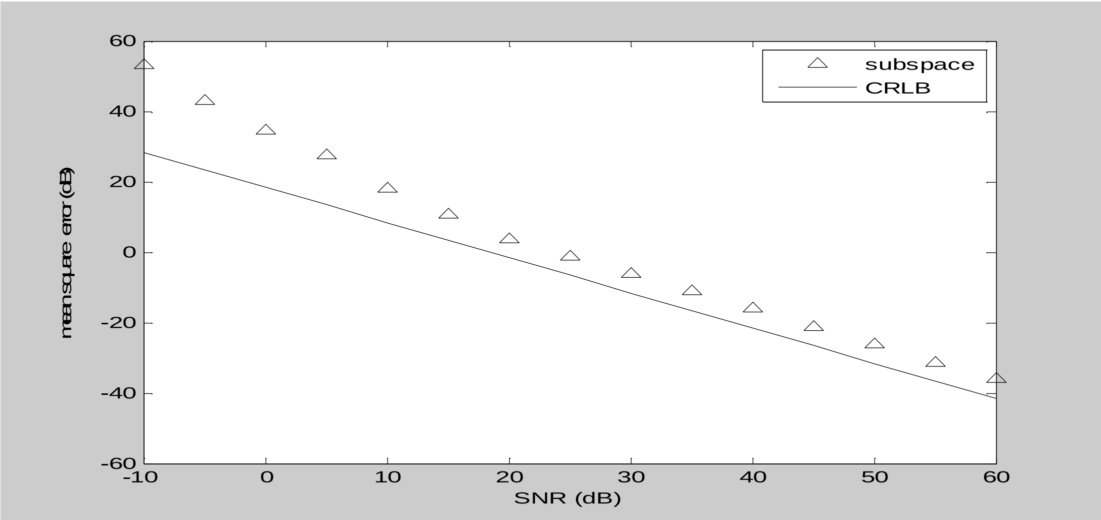


Fig. 3 MSE Performance versus SNR for Subspace and CRLB Blind Estimators

From Fig 3 it can observed that CRLB estimator exhibits high performance compared to subspace estimator. The performance of CRLB is exceptionally good especially at low SNR values.

Bayesian Cramer-Rao Lower Bounds (CRLB) is derived for analyzing and estimating phase noise. The offline and online BCRLBs are evaluated analytically using the closed-form expressions in [12] and [14]. The online (filtering) BCRLB is plotted for frame length K=20 and offline (smoothening) BCRLB is plotted for frame lengths K=[3; 5; 10; 15; 20]. Similarly the offline and online BCRLBs are evaluated by numerically evaluating the expectation of the elements of the Hessian matrix, or 1000 Monte-Carlo simulations. The online BCRLB is plotted for frame length K=20 and offline BCRLB is plotted for frame lengths K=[3; 5; 10; 15; 20]

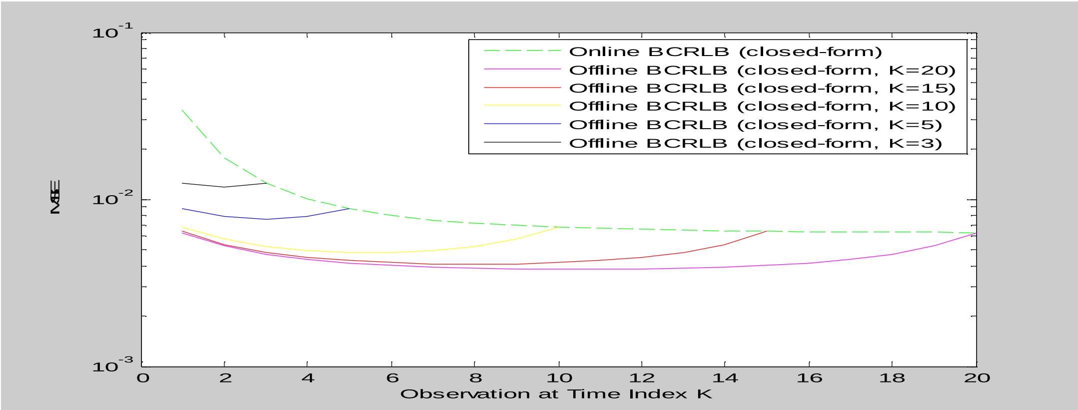


Fig. 4 Online and offline BCRLB versus the number of observations for 2 * 2 MIMO system with SNR = 5 dB using Analytical Approach

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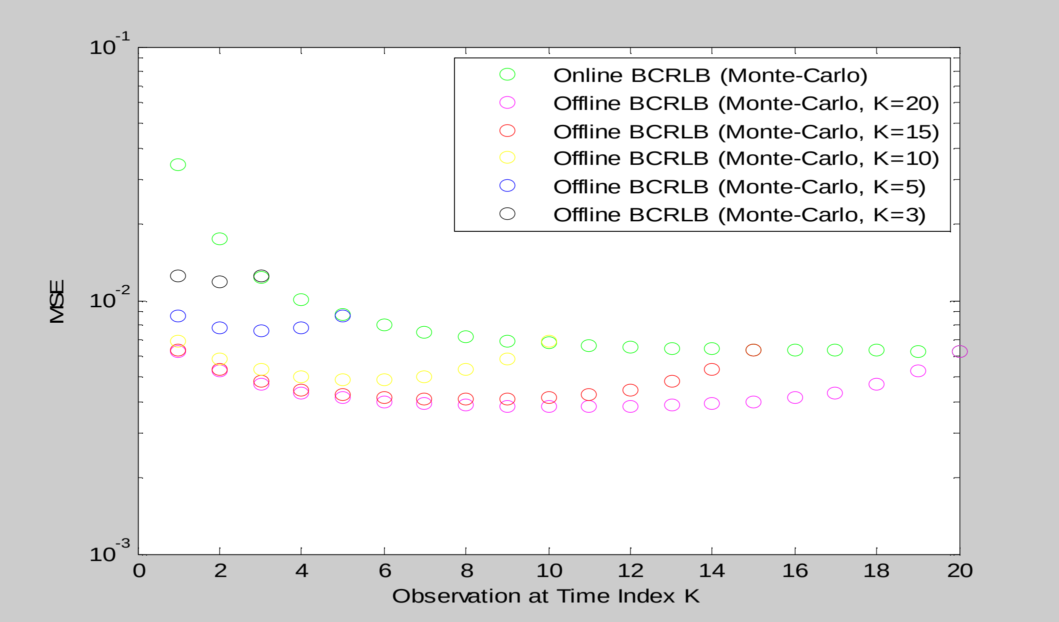


Fig. 5 Online and offline BCRLB versus the number of observations for 2 * 2 MIMO system with SNR = 5 dB using Analytical Approach

Fig 4 and Fig 5 show the Plot of the offline and online BCRLBs for frame lengths of K = [3; 5; 10; 15; 20] and a 2 x 2 MIMO system with an SNR of 1=2x2 w = 5 dB. The results in Fig 4 and Fig 5 reveal that the minimum and maximum values of the offline BCRLB are achieved at the middle and end-points of a frame, respectively, for any frame length, K. This implies that the best phase noise estimate can be achieved for the middle symbol within the frame, whereas the estimates get poorer as one move to the boundary points. This behaviour is expected since the phase noise for the symbol in the middle of the frame is followed by the largest number of past and future symbols with highly correlated phase noise values. Thus, by exploiting the observed symbols and correlations, the phase noise values corresponding to the middle symbol can be estimated with the highest accuracy. This can be observed from the time dependency of the BCRLB. Moreover Fig 4 and Fig 5 shows that the online BCRLB decreases with increasing observation length, K, since the longer the length of the observation sequence the more information is available for estimation of the $k^{th}$ symbol's phase noise

## V. CONCLUSIONS

Different types of channel estimators are simulated and studied for performance. It can be observed that the Cram´er-Rao Lower Bound (CRLB) estimator has superior performance in comparison to ML estimator especially in low SNR scenario. Even though the performance of ML estimator is comparable to that of Cram´er- Rao Lower Bound (CRLB) it can be concluded that in a blind channel estimation scenario CRLB gives us the best possible estimation of SNR. Similarly it can also be observed that CRLB maintains its superior performance over another blind channel estimation technique namely subspace coding across the entire spectrum of SNR from low to high. Also in this Bayesian Cram´er-Rao Lower Bound estimator for estimating phase noise which is very crucial in avoiding signal deterioration and improving signal quality is derived. The performance of BCRLB for both online and offline for 2* 2 MIMO System using both analytical and numerical approaches is evaluated. It can be concluded that a high degree of estimate can be achieved for phase noise of middle symbol within a frame and the estimates get poorer as moved towards the boundary.

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## ACKNOWLEDGEMENTS

The author’s would like to acknowledge the SSN College of Engineering and Technology, Ongole and University College of Engineering and Technology, Acharya Nagarjuna University, Guntur for providing a platform for doing this research work.

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