ISSN (Online) : 2319 - 8753
ISSN (Print) : 2347 - 6710

International Journal of Innovative Research in Science, Engineering and Technology

*An ISO 3297: 2007 Certified Organization, Volume 2, Special Issue 1, December 2013*

**Proceedings of International Conference on Energy and Environment-2013 (ICEE 2013)**

**On 12th to 14th December Organized by**

**Department of Civil Engineering and Mechanical Engineering of Rajiv Gandhi Institute of Technology, Kottayam, Kerala, India**

# FINITE ELEMENT ANALYSIS OF UNDER WATER TOWED CABLES

**Vandana RK**

PhD Scholar, College of Engineering, Trivandrum, India

**ABSTRACT**

The towing of an underwater object by means of a cable attached to a ship is an important problem in underwater technology. An underwater towed body is generally at the mercy of hydrodynamic and hydrostatic forces, which controls its motion through water. These forces affect the depth characteristics of the towed body giving specific configuration to the towline. This work consists of the development of a finite element program for the dynamic analysis of underwater towed cable and its tow characteristic

**NOMENCLATURE**

| | |
|---|---|
| $C_D$ | Drag Coefficient |
| $C_M$ | Inertia Coefficient |
| $C_a$ | Added Mass Coefficient |
| $C_x$,$C_y$,Cz | Direction cosines of the element |
| $F_G$ | Gravitational force acting on the member |
| $F_D$,$F_N$ | Normal and Tangential components of the drag force |
| $D_o$ | Total Drag on the towed body. |
| d | Water depth |
| L | wave length |
| Re | Reynod's Number |
| S | Scope of the Cable |
| T | Tension in the Cable at any instant |

## 1. INTRODUCTION

Different types of underwater vehicles, also called 'submersibles', are being used for exploration and use in ocean environment. Towed systems are attached to a surface vehicle and are being towed by means of electromechanical cables called tow cables. Towed systems find wide applications in military, oceanographic research, geological exploration and fisheries.

There are two basic purposes for which cables are used in the marine environment. They transmit power to the underwater equipment or between land points that are separated by water and carry signals for communication, control or telemetry purposes. In addition to the above, marine towing cables are capable of transmitting tension between the tow body and tow craft [1-2]. The cables that contain electrical conductors for signal or power transmission and strength members for supporting a mechanical load are commonly referred to as 'electromechanical' cables. The present work deals with the analysis of this type of cables.

## 2. UNDERWATER TOWED SYSTEMS

Underwater towed systems represent an overwhelming number of systems that have been towed behind ships and boats to perform many different types of works. The primary method of operation for towed systems is to launch the usually heavy vehicle and then tow it at the desired depth by varying the length of the strong electromechanical cable. Figure 1. Shows an underwater towed system with a multi part tow configuration.

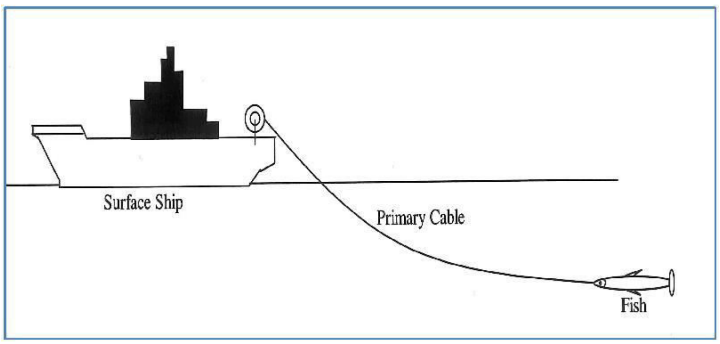



FIGURE 1. UNDERWATER TOWED SYSTEM

## 3. FORCES ACTING ON TOWCABLE

Generally, a towed cable is subjected to three types of forces. The hydrodynamic force generated by waves and currents, the hydro static force of buoyancy and the gravitational force. The buoyancy force calculation is however trivial. In the computation of wave loads Airy's linear wave theory is most widely used to obtain the wave kinematics. By using the well-known Morison's equation, wave force calculation is done in two phases: i) calculation of kinematics (velocity and acceleration) of flow field ii) relating the forces(inertia and drag) to the kinematics by means of the coefficients $C_D$ and $C_M$. The current force is calculated similarly as above by using the current velocity as the kinematics and setting the inertia term as zero[3-4].

In the present analysis, the non-linear characteristics of the hydrodynamic loading in terms of the relative motion between the structure and fluid have been incorporated. The drag force can be separated into normal and tangential components. These forces and gravity forces are referred to per unit length of the member. The normal drag on the cable ($F_N$) is caused by the component of the velocity normal to the element and the tangential drag ($F_D$) is taken as a fraction of the normal drag (2 to 3%). Tangential drag is neglected in this analysis. The gravity force is given by $F_G$=W-B where W and B are the weight and the buoyancy per unit length of the member. It should be noted that the effective velocity should be obtained by superimposing vectorially the ship's velocity and the existing wave velocity acting on the cable. The effect of lift forces due to shape of the cable cross-section, vortex shedding induced load etc. are ignored[5-6].

### 3.1 Derivation of Wave Forces

The modified form of Morison's equation (including the fluid-structure interaction effect) for calculating the drag and inertia forces on an element dz along the length of the cable becomes,

$$\delta F_d = \frac{1}{2} D\rho_f \; C_D \, |u - \dot{x}| \, (u - \dot{x}) \mathrm{dz} \qquad (1)$$

$$\delta F_I = \frac{\pi}{4} \, D^2 \, \rho_f \, [C_M \, \dot{u} - (C_M - 1) \, \ddot{x}] \, dz \qquad (2)$$

The vector representation of the Morison's equation is

$$\hat{f} = \frac{\pi}{4} D^2 \rho_f C_M \widehat{\dot{\omega}} + \frac{1}{2} D \rho_f C_D \, \widehat{\omega} \, |\widehat{\omega}| \qquad (3)$$

Where $\hat{f}$ is the force vector per unit length; $\widehat{\omega}$ and $\widehat{\dot{\omega}}$ are the normal velocity and acceleration vectors, respectively; $|\widehat{\omega}|$ is the magnitude of the velocity vector. The normal component is composed of horizontal and vertical water particle velocity and acceleration components, while the tangential component is ignored. In a rectangular Cartesian co ordinate system,if I,j,k are the unit vectors along y and z directions and if $C_X$, $C_Y$ and $C_Z$ are the direction cosines along the three directions respectively, then, the

$$\begin{Bmatrix} \widehat{\omega} \\ \widehat{\dot{\omega}} \end{Bmatrix} = \begin{Bmatrix} u_{nx} \\ \dot{u}_{nx} \end{Bmatrix} \hat{\imath} + \begin{Bmatrix} u_{ny} \\ \dot{u}_{ny} \end{Bmatrix} \hat{\jmath} + \begin{Bmatrix} u_{nz} \\ \dot{u}_{nz} \end{Bmatrix} \hat{k} \qquad (4)$$

In which

$$\begin{Bmatrix} u_{nx} \\ u_{ny} \\ u_{nz} \end{Bmatrix} = [T] \begin{Bmatrix} u \\ v \\ 0 \end{Bmatrix} \; and \; \begin{Bmatrix} \dot{u}_{nx} \\ \dot{u}_{ny} \\ \dot{u}_{nz} \end{Bmatrix} = [T] \begin{Bmatrix} \dot{u} \\ \dot{v} \\ 0 \end{Bmatrix} \qquad (5)$$

Where

$$[T] = \begin{bmatrix} 1 - {C_x}^2 & -C_x C_y & -C_x C_z \\ -C_x C_y & 1 - {C_y}^2 & -C_y C_z \\ -C_x C_z & -C_y C_z & 1 - {C_z}^2 \end{bmatrix} \qquad (6)$$

The dot over each term represents the respective derivative with respect to time [7-8]. By back substitution, the non linear force per unit length is obtained as

$$\begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} = \rho_f V_{ol}[T]\begin{Bmatrix} \dot{u} \\ \dot{v} \\ 0 \end{Bmatrix} + \rho_f C_a V_{ol}[T]\begin{Bmatrix} \dot{u}-\ddot{x} \\ \dot{v}-\ddot{y} \\ 0-\ddot{z} \end{Bmatrix} +$$

$$\frac{1}{2}\rho_f C_D A[T]\begin{Bmatrix} u-\dot{x} \\ v-\dot{y} \\ 0-\dot{z} \end{Bmatrix}|u_{nr}| \qquad (7)$$

Where

$$[u_{nr}] = \left((u_{nx}-\dot{x})^2 + \left(u_{ny}-\dot{y}\right)^2 + (u_{nz}-\dot{z})^2\right)^{\frac{1}{2}} \qquad (8)$$

And

x, y,z and $\dot{x}$, $\dot{y}$, $\dot{z}$ are the structural responses and velocities respectively.

Where`

$$u = \pi\frac{H}{T}\frac{\cos hk(d+y)}{\sin hk(d+\eta)}\cos(kx-\omega t) + V$$

$$v = \pi\frac{H}{T}\frac{\sin hk(d+y)}{\sin hk(d+\eta)}\sin(kx-\omega t)$$

$$\dot{u} = 2\pi^2\frac{H}{T^2}\frac{\cos hk(d+y)}{\sin hk(d+\eta)}\sin(kx-\omega t)$$

$$\dot{v} = -2\pi^2\frac{H}{T^2}\frac{\sin hk(d+y)}{\sin hk(d+\eta)}\cos(kx-\omega t) \qquad (9)$$

And V is the towing speed.

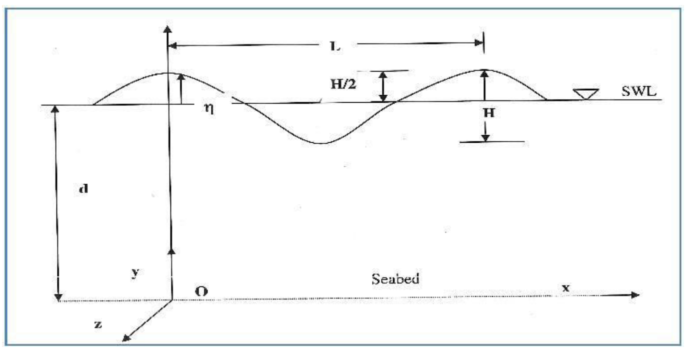



FIGURE 2.DEFINITION SKETCH FOR WATER PARTICLE KINEMATIC EXPRESSIONS

## 4. FINITE ELEMENT FORMULATION

Non linear structural behavior has two principal sources, namely material non linearity and geometric non linearity. When a ship is towing a towed body with the help of a cable, the cable body system is subjected to current loading leading to large displacements of the cable. This essentially is a geometric non linear problem in practice, since the cable material is such that under no operating condition a material non linearity does set in as the strains remain small. Updated Lagrangian formulation is used to analyze this large displacement problem. In this formulation, a local coordinate system is attached to each element. The local system moves along with the element. Differentiations and integrations are done with respect to this system. The current deformed state is used as the reference state prior to the next step of the solution. Then the coordinates are updated to produce a new reference state [9-10].

### 4.1 Finite Elements

Generally cable structures are modeled by bar elements, as resistance to bending is often negligible. But in this work, a beam element is chosen to model the cable, the reason being that one can estimate the geometric non linear effects more accurately with a beam model as well as cater to a cable segment where the bending rigidity may not be negligible [11].

### 4.2 Equations of Equilibrium

The dynamic equations of equilibrium for a linear multi degree freedom system is given by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{R\} \qquad (10)$$

Where [M],[C] and [K] are mass, damping and stiffness matrices_ respectively;{R}is the external load vector;{ },{ },{ } are the displacement, velocity and acceleration vectors of the full structure. The terms [M]{ },[C]{ },[K]{x} represent the inertia, damping and elastic forces respectively at a time ‘t’. Equation (10) represents a system of linear differential equation of second order with constant coefficients. This equation can be viewed in the finite element context as the equations of equilibrium of a finite element assemblage, which in this work is made up of three-dimensional beam element as shown in Figure 3.

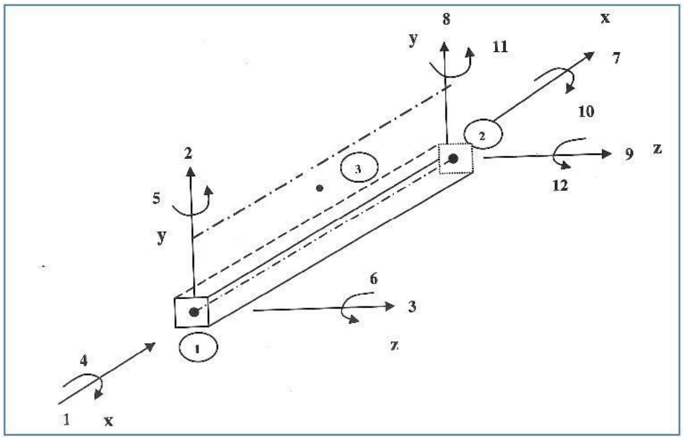



FIGURE 3. SCHEMATIC VIEW OF THE 3D BEAM ELEMENT IN LOCAL CO-ORDINATE AXES

**4.3 Mass and Added Mass matrices** The total mass is the sum of physical mass and added mass. The consistent mass matrix for the 3D beam element was used. Added mass in the lateral directions for the beam element was calculated from beam shape functions.

**4.4 Damping Matrix** The drag term of Morison equation has taken care of the hydrodynamic damping. Structural damping was included in the form of Rayleigh damping.

**4.5 Stiffness Matrix** The linear stiffness matrix [12×12] for the 3D prismatic beam elements was used for this study. The member stiffness matrix can be transformed to the global system for an arbitrarily oriented 3D beam element using a transformation matrix.

**4.6 External Load Vector** The wave particle velocities and accelerations were calculated in the global direction and from these values, normal component of the velocity and acceleration were calculated. The wave forces were calculated then for a randomly oriented element in space using Morison equation. Wave force, self weight and buoyancy force were combined together and converted into equivalent nodal loads by using Gauss Numerical Integration [12-13].

## 5. PROGRAM DEVELOPMENT AND VALIDATION

A general purpose program was developed for the dynamic analysis of the tow cable using finite element method. The program is able to analyze any structural system which can be discretized into 3D beam elements. The program was developed in C. In order to validate the reliability of the software, a typical case study was selected [14]. The data taken from selected case study was fed to the program and the results were compared with the reported ones. The response characteristics as those of the reported results validated the present program.

## 6. NUMERICAL CASE STUDY

The steady state analysis has been conducted for an underwater towed system consisting of a primary tow cable (medium and heavy) and a towed body (vertical elliptical cylinder). The physical properties are shown in Table 1. and Table 2[15].

TABLE 1. PHYSICAL PROPERTIES OF TOW FISH

| Tow body | Dimension(mm) | Frontal area(m2) | Weight in air | Weight in water | Body drag coef.(CDo) |
|---|---|---|---|---|---|
| Vertical Elliptical cylinder | Major axis-600 Minor axis-400 Height-550 | 0.22 | 165 | 58.7 | 0.301 |

TABLE. 2. PHYSICAL PROPERTIES OF THE TOW CABLE

| Tow Cable | Diam-eter (mm) | Weight in air (kg/m) | Weight in water (kg/m) | Density ($Kg/m^3$ ) |
|---|---|---|---|---|
| Medium | 20 | 1.068 | 0.745 | 3400 |
| Heavy | 32 | 4.02 | 3.195 | 4900 |

It is important to note that the body was not discretized into finite elements. The tension in the body is calculated as the resultant of its weight and the hydrodynamic resistance. Two different types of cables are selected in order to study its effect on tow performance curves. The speed of the ship is varied from 5 knots to a maximum speed of 30 knots. The maximum tow speed is of the order of 25 to 30 knots, which is called survival speed in tactical applications.

## 7. RESULTS AND DISCUSSIONS

### 7.1 MediumCable

The problem definition sketch of the towed cable body is shown in Fig. 4. The profile generated for medium cable for different towing speed is shown in Fig. 5. Operating depth is found to be high when the speed is less, especially for 5 knots. For all the other speeds the profile is beginning to be horizontal when the scope is in the range of 200m to 300m. i.e., the speed does not have much influence on the profile after this range.

**Dynamic Tension** Tension is plotted with respect to time for different velocities. The variation of dynamic tension for speeds 10 and 25 are shown in Fig.6. and Fig.7.

### 7.2 Heavy Cable

Fig. 8. Shows the profile generated for heavy cable at different towing speeds varying from 5 knots to 30 knots. Operating depth is found to be high when the speed is 5 knots. For all the other speeds, the profile is beginning to be horizontal when the scope is in the range of 200m to 300m.

**7.3 Dynamic Tension** The variation of dynamic tension for speeds 10 and 25 are shown in Fig. 9.and Fig. 10.

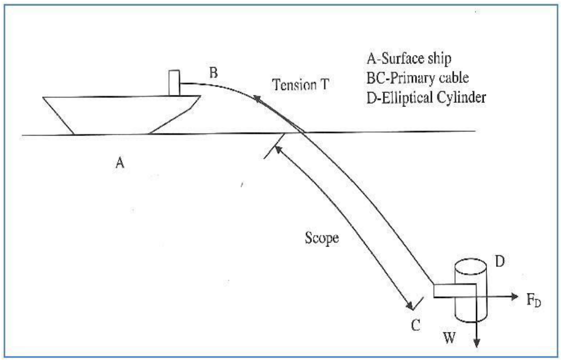
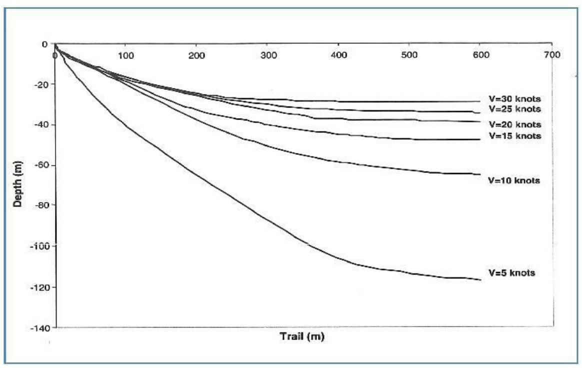
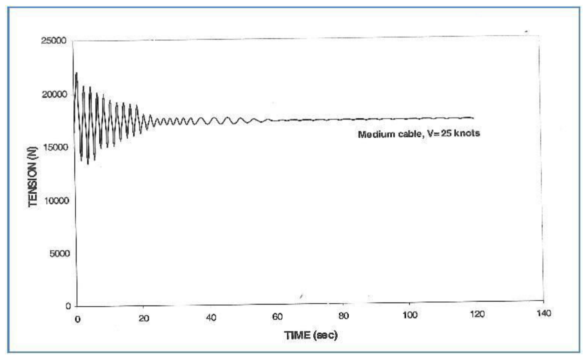


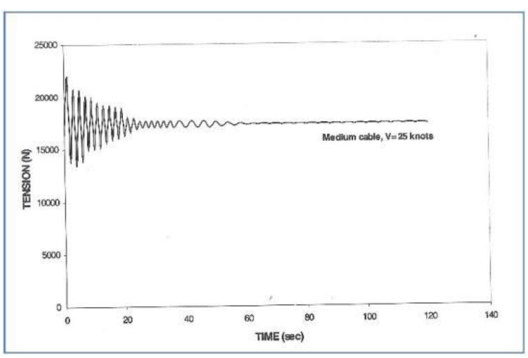

FIGURE 4. DEFINITION SKETCH OF
CABLE BODY SYSTEM FIGURE




5. PROFILE FOR MEDIUM CABLE

FIGURE 6.TENSION Vs TIME PLOT(Medium Cable, 10 knots)



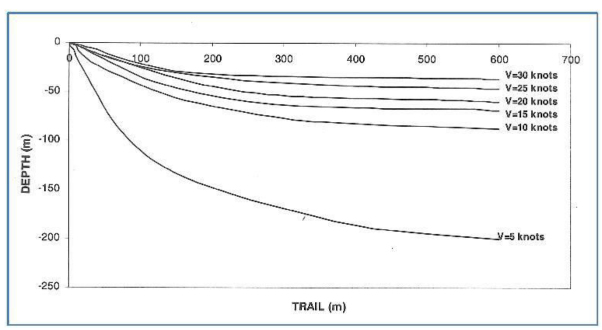




FIGURE 7. TENSION Vs TIME PLOT(Medium Cable,25 knots)

FIGURE 8. PROFILE FOR HEAVY CABLE

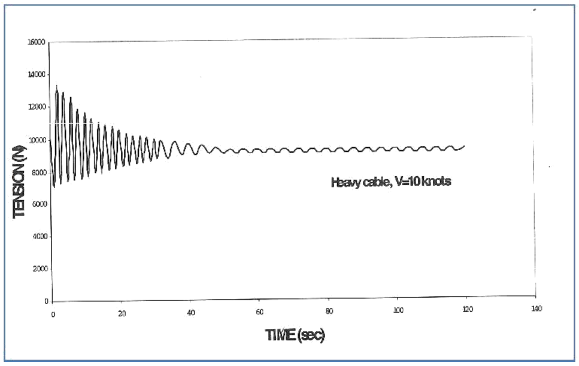



FIGURE 9. TENSION Vs TIME PLOT(Heavy Cable, 10 knots)

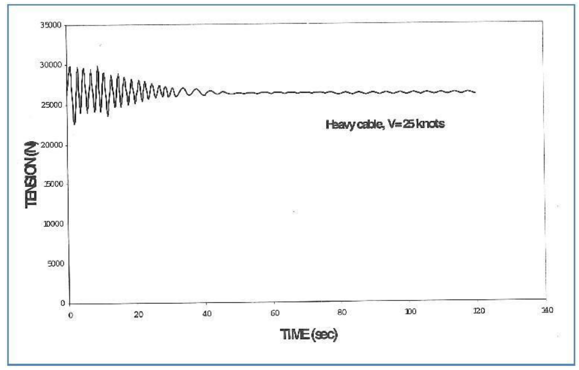



FIGURE 10. TENSION Vs TIME PLOT(Heavy Cable, 25 knots)

Table 3. shows the comparison of cable tension at two different towing speeds.

TABLE 3. TENSION VALUES OF MEDIUM AND HEAVY CABLES

| Velocity (knots) | Max. Transient Tension (N) | | Steady-state Tension(N) | | Static Tension[15] (N) | |
|---|---|---|---|---|---|---|
| | Medium Cable | Heavy Cable | Medium Cable | Heavy Cable | Medium Cable | Heavy Cable |
| 10 | 6760.55 | 13345.45 | 3409.28 | 9181.805 | 3233.9 | 7363.09 |
| 25 | 21989.76 | 29834.56 | 17344.362 | 26265.84 | 15005.6 | 24284.57 |

## 8. CONCLUSIONS and FUTURE RESEARCH

Given the physical and hydrodynamic properties of the towed body and tow cable, the profiles were generated for two different types of cable. The operating depth of the towed body is found to decrease as the ship's speed increase. From the dynamic analysis of the cable-body system, it is observed that the value of tension is high compared to the reported results obtained for static analysis. i.e., the tension values are under estimated in static analysis. The value of tension is very important for the design and selection of towing cable. This underlines the necessity of conducting the dynamic analysis for the underwater towed systems.

There is much room for extending the application of the finite element method to cable dynamics problems. An obvious extension would be to the treatment of 3D towing problems considering a turn of the towing vehicle. Departing from the towing problem, the method may have application in the fields of mooring and cable laying.

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