Research & Reviews: Journal of Statitics and Mathematical science

Fuzzy Incident Matrix

P. Divya, C. Sindhujaa

Department of Mathematics, Nadar Saraswathi College of Arts and Science, Vadaputhupatti, Theni District, Tamilnadu, India.

Research Article

Received date: 21/04/2021
Accepted date: 25/06/2021
Published date: 30/06/2021

***For Correspondence**

C. Sindhujaa, Department of Mathematics, Nadar Saraswathi College of Arts and Science, Vadaputhupatti, Theni District, Tamilnadu, India

E-mail: ranjansindhu98@gmail.com

Keywords: Incident matrix, Properties of matrix, Types of matrices

ABSTRACT

In this dissertation, the new concept, "fuzzy incident matrix" is defined. Theorems of fuzzy incident matrices are proved from the fuzzy adjacent matrices. By using two binary operators the properties of matrices are calculated and proved and also the fuzzy incident matrix is proved for the types of matrices.

INTRODUCTION

The matrix has a long history of application in solving linear equations. They were known as arrays until the 1800's. The term matrix (Latin for "womb", derived from mater - mother) was coined by James Joseph Sylvester in 1850, who understood a matrix as an object giving rise to a number of determinants today called minors, that is to say, determinants of smaller matrices that are derived from the original one by removing columns and rows. An English mathematician named Cullis was the first to use modern bracket notation for matrices in 1913 and he simultaneously demonstrated the first significant use of the notation A = a_{ij} to represent a matrix where a_{ij} refers to the element found in the i^{th} row and the j^{th} column. Matrices can be used to compactly write and work with multiple linear equations, referred to as a system of linear equations, simultaneously. Matrices and matrix multiplication reveal their essential features when related to linear transformations, also known as linear maps. A matrix is a rectangular, arrays of numbers, symbols, or expressions, arranged in rows and columns. Matrices are commonly written in box bracket. The horizontal and vertical lines of entries in a matrix are called rows and columns. The size of a matrix is defined by the number of rows and columns that it contains. A matrix with m rows and n columns is called as m × n matrix or m – by–n matrix, while m and n are called it dimension. Fuzzy matrices were introduced for the first time by Thomason, who discussed the convergence of powers of fuzzy matrix. Fuzzy matrices play a vital role in scientific development. A fuzzy matrix may be a matrix with elements having values in the fuzzy interval. Fuzzy matrices occur within the modeling of assorted fuzzy systems, with products usually determined by the "max(min)" rule. Fuzzy matrices are of fundamental importance in the formulation and analysis of many classes of discrete structural models which arise in physical, biological, medical, social and engineering sciences. Fuzzy matrices require a completely separate treatment form that of matrices over the real field or complex field, due to which fuzzy matrices do not satisfy many of the fundamental properties of real or complex matrices.

In this paper we have discussed about the theorems of fuzzy incident matrix from adjacent matrix and properties of matrices are proved for fuzzy incident matrix and it is proved for nilpotent matrix.

PRELIMINARIES

Definition: 2.1

Let S be a nonempty set, then a fuzzy subset of S is a map s from S into the real interval [0, 1]. We say that a fuzzy subset is a crisp if s(x) is in {0, 1} for every x in S.

Definition: 2.2

A fuzzy graph ü üü ℵ is a triple consisting of a nonempty set V together with a pair of functions

ℵ üü and ℵℵ üü such that for all x, yϵV,

üüüüüℵℵ

Definition: 2.3

Let üü ℵ be a fuzzy graph. The degree of a vertex u is üüüüℵ .

Definition: 2.4

The complement of a fuzzy graph üü ℵ is a fuzzy graph ℵ , where ℵ ‾ and

‾ℵüüüüüℵℵ

Definition: 2.5

A fuzzy matrix is the matrix whose elements are taking their values from [0,1].

0.1	0	0.2
0.5	0.4	0.3
0.2	0.3	0.1

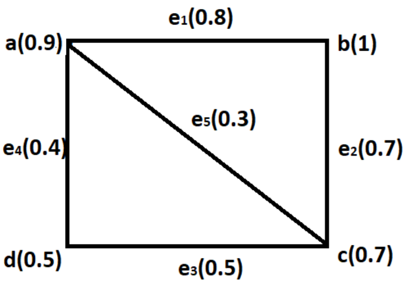
Definition: 2.6

If A = $[a_{ij}]$ and B = $[b_{ij}]$ are fuzzy matrices of order n × p, their addition is defined as A+B = $[a_{ij} \vee b_{ij}]$ where V stands for the max multiplication of fuzzy matrices is done by the maxmin operation

Definition: 2.7

The fuzzy incident matrix of fuzzy graph is defined as

$ü_{ii}$ üü $\frac{ü_{ii}üüü=üüüüüüüü}{ßüüüü}$



The fuzzy incident matrix of the above fuzzy graph is given by

ü ℵüüüüü ℵüüüü ℵüüüüü ℵüüüü

RESULTS ON FUZZY INCIDENT MATRIX

Each column representing a branch contains two non-zeros entries positive and negative; the rest being zero.

The entries in each rows shows that the total values of the edges for each vertices and it represent the values of the edge.

Given the fuzzy incident matrix A the corresponding graph can be easily constructed since A is a complete mathematical replica of the graph

If one row of a is deleted the resulting (n-1)×b matrix is called the reduced fuzzy incidence matrix A_1. Given A_1, A is easily obtained by using the first property

The diameter of the incident matrix is not zero. It will be a unit matrix.

Theorem 3.1

The fuzzy graph corresponding to fuzzy incident matrix is not complementary and self-complementary.

Proof:

Let (σ, μ) be the fuzzy graph corresponding to the complement fuzzy incidence matrix.

üüüüüℵℵ .

ℵℵüüüüüüüℵ and üüüüüℵ ╱ ℵü

Hence Fuzzy graph is not complementary and self – complementary.

Theorem 3.2

The fuzzy incident matrix of every undirected fuzzy graph G (σ, μ) is not symmetric.

Proof:

Let üüℵ be the undirected simple fuzzy graph

ℵ üüüüüü

Let $I_{FG} = (a_{ij})$ be the fuzzy incident matrix of ℵ üüüü ü

The membership values of I_{FG} of any row and column are different

ℵ üüüü ü

ü is not symmetric

Theorem 3.3

If I(G) is an fuzzy incident matrix of a connected graph G with n vertices, the rank of I(G) is n-1

Proof:

If we remove any one row from the fuzzy incidence matrix of a connected graph, the remaining (n - 1) by e submatrix is of rank n - 1.

In the other words, the remaining n – 1row vectors are linearly independent. Thus we need only n – 1rows of an fuzzy incidence matrix to specify the corresponding graph completely, for n - 1 rows contain the same amount of information as the entire matrix.

Such as (n – 1) by e sub matrix A_f of A is called a reduced fuzzy incidence matrix. The vertex corresponding to the deleted row in A_f is called the reference vertex. Clearly, any vertex of a connected graph can be made the reference vertex.

PROPERTIES OF MATRICES

Commutative in addition

A + B = B + A

The above condition for commutative in addition is proved for fuzzy incident matrix.

Commutative in multipication

A × B = B × A

The above condition for commutative in multiplication is proved for fuzzy incident matrix.

Associative in addition

A + (B + C) = (A + B) + C

The above condition for associative in addition is proved for fuzzy incident matrix.

Associative in multiplication

A×(B×C) = (A×B) ×C

The above condition for associative in multiplication is proved for fuzzy incident matrix.

Absorption

Absorption in both addition and multiplication is not proved for the fuzzy incident matrix.

Distributive

Distributive in both addition and multiplication is not proved for the fuzzy incident matrix.

Nilpotent matrix:

The determinant and trace of a nilpotent matrix are always zero. The only nilpotent diagonalizable matrix is the zero matrixes. Every singular matrix can be expressed as a product of nilpotent matrices

The fuzzy incident matrix can be written in each type of the matrix (e.g. row matrix, column matrix, square matrix etc.). Meanwhile the fuzzy incident matrix can be proved for null matrix.

CONCLUSION

In this dissertation the definition and the concepts of, "Fuzzy Incident Matrix" and using the definition theorems are proved from the adjacency matrix. Also, the properties of matrices such as commutative in both addition and multiplication, associative in addition and multiplication, absorption and distributive are calculated and also it is proved with nilpotent matrix.

Some properties are calculated for connected graph and some are calculated for complete graph. It can be proved for some types of matrices and it can be proved for connected or complete graph.

REFERENCES

1. Diaz, J. A. Solving multi-objective transportation problem. EKON.-MAT. OBZ. 1978;14: 267-274.
2. Diaz, J. A. Finding a complete description of all efficient solutions to a multi-objective transportation problem. EKON.-MAT. OBZ. 1979; 15:62-73.
3. Isermann, H. The enumeration of all efficient solutions for a linear multi-objective transportation problem. *Nav. Res. Logist. Q.*1979; 26:123-139.
4. Ringuest, J. L., Rinks, D. B. Interactive solutions for the linear multi-objective transportation problem. *Eur. J. oper. Res.*1987;32:96-106.
5. Zimmermann. H. –J. Applications of fuzzy set theory to Mathematical programming, Inform. Sci. 1985;34:29-58.
6. Bit, A. K. Fuzzy programming with Hyperbolic membership functions for Multi-objective capacitated transportation problem. OPSEARCH. 2004;41:106-120.
7. F. L. Hitchcock. The distribution of a product from several sources to numerous localities. *J. Math. Phys.* 1941;20:224-230.
8. L. A. Zadeh. Fuzzy Sets, *Inf. Control*. 1965;8:338-353.
9. R. E. Bellman, L. A. Zadeh. Decision making in a fuzzy environment, *Manag. Sci.* 1970;17: B141-B164.
10. R. Verma, M. P. Biswal and A. Biswal. Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions, *Fuzzy Sets Syst.* 1997;91:37-43.
11. L. Li and K. K. Lai. A fuzzy approach to the multiobjective transportation problem, *Comput. Oper. Res.* 2000;27:43-57.
12. Abd F. Waiel Wahed. A multi-objective transportation problem under fuzziness. *Fuzzy Sets Syst.* 2001;177:27-33.
13. Leberling, H. On finding compromise solution for multi-criteria problems using the fuzzy min-operator. *Fuzzy sets syst.*1981;6:105-118.
14. Dhingra, A.K., Moskowitz, H. Application of fuzzy theories to multiple objective decision making in system design. *Eur. J. Oper. Res.*1991;55:348-361.
15. Verma Rakesh, Biswal, M.P., Biswas, A. Fuzzyprogramming technique to solve Multi-objectivetransportation problems with some nonlinear membership function. *Fuzzy sets syst.*, 1997;91:37-43.
16. R. J. P. Atony, Savarimuthu SJ, Pathinathan T. Method for solving the transportation problem using triangular intuitionistic fuzzy number. *Int. J. Comput. Algorithm.* 2014;03: 590-605.
17. S. K. Singh, S.P. Yadav. A new approach for solving intutionistic fuzzy transportation problem of type-2, *Annu. Oper. Res.* 2014;243:34-363.
18. P. S. Kumar, R. J. Hussain. Computationally simple approach for solving fully intuitionistic fuzzy real life transportation problems. *Int. J. Syst. Assur. Eng. Manag.* 2015;1:1-12.
19. Sonia and Rita Malhotra. A Polynomial Algorithm for a Two Stage Time Minimizing Transportation Problem. Opsearch. 2003;39: 251-266.
20. Nagoor Gani, K. Abdul Razak. Two Stage Fuzzy Transportation Problem, *J. sPhysical Sci.* 2006;10:63-69.
21. R. J. LI and E. Stanley Lee. An Exponetial Membership Function for Fuzzy Multiple-objective Linear Programming. *CompatcrJ Math, Applic.* 1991;22:55-60.
22. S. K. Das, A. Goswami and S. S. Alam. Multiobjective transportation problem with interval cost, source and destination parameters, *Eur. J. Oper. Res.* 1997;117:100-112.