ISSN (Print) : 2320 – 3765

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 9, September 2016

Mathematical Modeling of Industrial Chains

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ABSTRACT: Each time one designs an automatic mechanism, the choice of the size of the electrical motors which are regarded as the main elements of any electromechanical system may be difficult. All technical and economic indices depend on the adequate size choice and the optimality of the operation modes of these motors. To be able to effectively undertake the study of the motorization of a complex industrial mechanism, the development of a simplified model is important for dimensioning and selecting the appropriate motor. This paper proposes a comprehensive and complete approach for the development of simplified models and the mathematical formulation of the dynamic equations of movements in the complex industrial mechanisms. These simplified models can be easily solved by numerical methods and their solutions can be used in the size selection. In this paper, the models that have been developed are valid for various applications starting from a simple mechanism with only one degree of freedom to the complex manipulator arms of the robots.

KEYWORDS: electrical motors, industrial mechanism, the dynamic equations, sizing.

I.INTRODUCTION

An industrial mechanism is constituted by the set of connected masses which are in rotation and / or translation movement. The development of the dynamic model of the mechanism, which represents all the moving masses and the links among them, is one of the essential steps for the analysis and design of complex mechanisms.

The following figure (Figure 1) provides an overview of the general structure of an industrial mechanism.

The electric motor develops torque T_m to put the various mechanism elements in rotation; the element D (Drum) will transform the rotary movement into a translational movement.

The load that has the mass m_k will move at the velocity (speed) $v_{load} = v_K$ under the effect of a force $F_{load} = F_K$.

Between the i^{th}mass J_i and $i^{th}+1$, there is a mechanical link with rigidity $h_i$which can be calculated from Hooke's law by the formula:

$$h_i = {}^{T_{ei}}\!/_{\Delta\varphi_i} \dots\dots\dots\dots \qquad (1)$$

Where:

T_{ei}: The load of elastic links.

$\Delta\varphi_i$: is the Deformation.

Similarly for the masses in translation motion; the rigidity can be calculated using Hooke's law:

$$h_j = {}^{F_{ej}}\!/_{\Delta S_j} \dots\dots\dots\dots \qquad (2)$$

Where:

F_{ej}: is the load.

ΔS_j: The longitudinal deformation.

h_j : The mechanical rigidity of the link between the masses m_j and m_{j+1}.

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II. MODELING OF THE MECHANISM

Significant influence on the dynamics of the mechanism is due to the large masses and the most elastic links. For this reason, one of the first key tasks for the design and the study of complex mechanisms with its driven motor is the development of a simplified model. The modelling is based on some assumptions such as:
- neglecting the elasticity of the mechanical links which are sufficiently rigid and the influence of small masses.
- different elements have not the same speeds; this prevent from direct comparison of their moments of inertia, masses, rigidities and displacements. Therefore, for the development of a computational scheme it is necessary to return all elements parameters such as speeds to a single equivalent speed. By convention, the speed of the electrical motor is taken as equivalent speed. The matching condition of the parameters conversion is the fulfillment of the energy conservation law.

Thus, during the conversion of the moment of inertia of a rotating system element with rotational speed ω_i (or the mass which moves with a translational speed $\boldsymbol{v_j}$ into the rotational speed of the motor shaft, the following condition may be satisfied:

$$(E_{ki})_r = J_{ri}\frac{\omega_1^2}{2} = E_{ci} = J_{ri}\frac{\omega_i^2}{2} \quad (3)$$

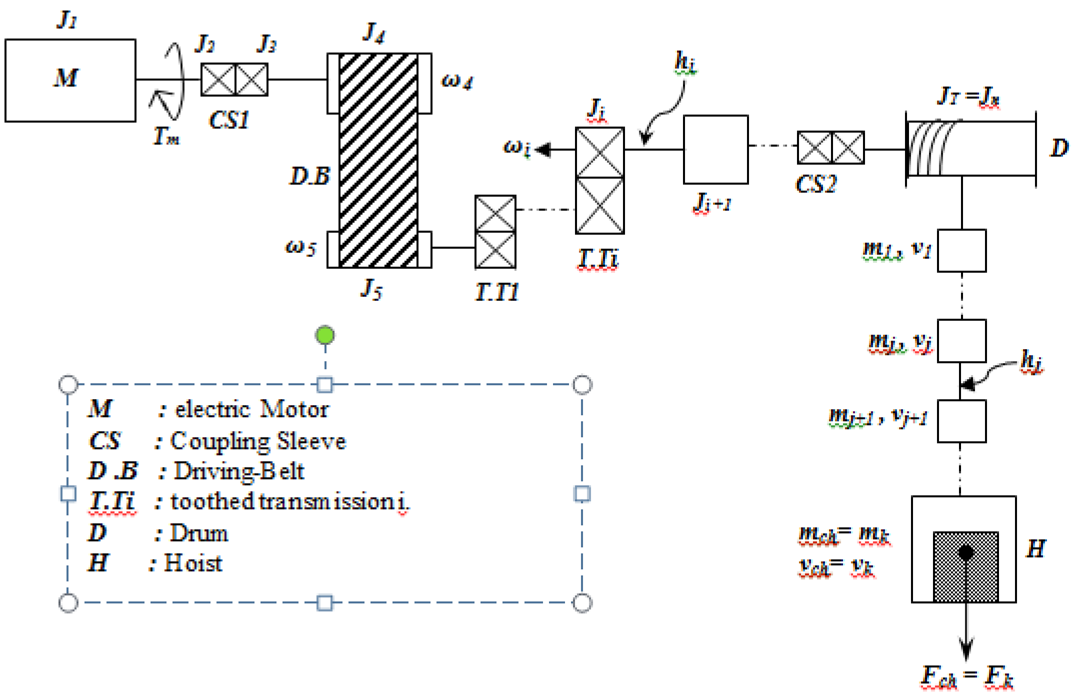

Figure 1: kinemato-dynamic scheme of an industrial mechanism.

Then,

$$J_{ri} = J_i\left(\frac{\omega_i}{\omega_1}\right)^2 = \frac{J_i}{i_{1i}^2} \quad (4)$$

Let i_{1i} is reduction coefficient which is expressed as follows:

$$i_{1i} = {}^{\omega_1}/_{\omega_i}\ {}^{\omega_1}/_{\omega_i} \quad (5)$$

While, in the case of a mass is in translation motion, the following condition may be satisfied:

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$$\left(E_{kj}\right)_r = J_{rj}\frac{\omega_1^2}{2} = E_{cj} = m_{rj}\frac{v_j^2}{2} \qquad (6)$$

Thus,

$$J_{rj} = m_j\left(\frac{v_j}{\omega_1}\right)^2 \qquad (7)$$

By introducing the reduction radius $\rho_{1j} = \frac{v_j}{\omega_1}$, Eq. (7) may be rewritten as follows:

$$J_{rj} = m_j\rho_{1j}^2 \qquad (8)$$

2.1. Displacements conversion:

From equations (5) and (7), one can deduce the displacements conversion formulas. Hence, for the rotating masses with

$\omega_1 = \frac{d\varphi_r}{dt}$;.... $\omega_i = \frac{d\varphi_i}{dt}$.

Then,

$$d\varphi_{ri} = i_{1i}\,.d\varphi_i \qquad (9)$$

If the speed of the masses which are in translational motion is $v_j = \frac{dS_j}{dt}$ then, the obtained displacement is given by:

$$d\varphi_{rj} = \frac{dS_j}{\rho_{1j}} \qquad (10)$$

The rigidities are:

$$h_{ri} = {h_i}/{i_{1i}^2} \qquad (11)$$

$$h_{rj} = h_j\,.\,\rho_{1j}^2 \qquad (12)$$

The forces and torques are given as:

$$T_{ri} = {T_i}/{i_{1i}} \qquad (13)$$

$$T_{rj} = F_j\,.\,\rho_{1j} \qquad (14)$$

2.2. Initial Computational Scheme:

To develop an equivalent calculation scheme, one should carry on a simplified initial scheme, in which the masses are represented by rectangles with their areas proportional to the moment of inertia.

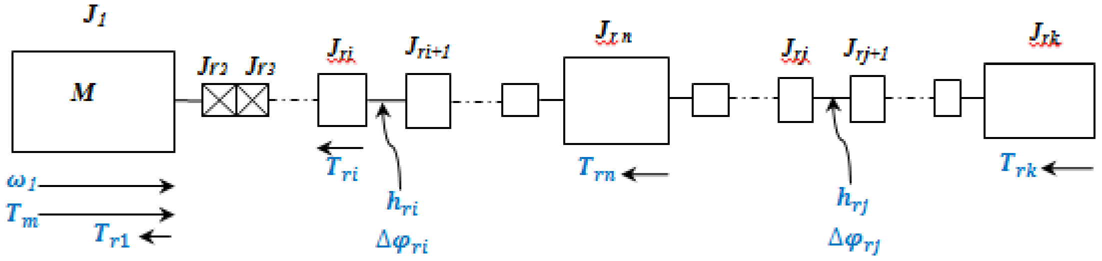

Figure 2: Initial computational scheme.

ISSN (Print) : 2320 – 3765

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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The rigidities of elastic links are represented by lines joining the masses whose lengths are inversely proportional to the values of these rigidities. On the initial pattern obtained (see Figure 2), one can distinguish three principal masses: the mass of the motor, the mass of the drum and the mass of the load.

2.3. Equivalent Computational Scheme:

The initial computational scheme is simplified by grouping the small masses to the closer large one.
In the obtained simplified scheme, the equivalent elasticity of the combined masses can be calculated from the following formula:

$$\frac{1}{h_{req}} = \frac{1}{h_{r1}} + \frac{1}{h_{r2}} + \frac{1}{h_{r3}} + \dots \dots \qquad (15)$$

To develop a simple and flexible mathematical model, the number of equations may be minimized. This depends on the structure and mechanism (rigid or elastic), the technical requirements of construction as well as the considered simplified assumptions. The initial computational scheme can be converted into the equivalent circuit diagram of three masses, twin mass or a single mass.
For example, the equivalent scheme of the initial computational scheme shown in Fig .2 can be represented as illustrated in Figure 3.

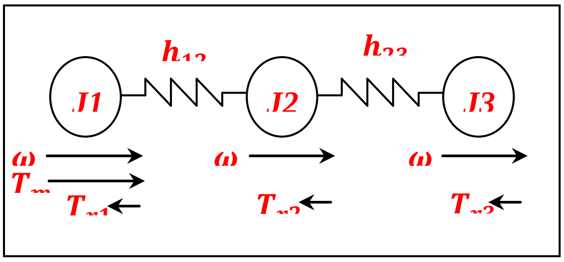

Figure 3: Equivalent scheme with 3 masses

If the elasticity between J_1 and J_2 (or J_2 and J_3) is neglected, the equivalent scheme becomes a two masses model where the masses and torques are added by taking into account their signs.

If the system is considered absolutely rigid, then, it can be converted into the equivalent scheme of a single mass. In this case, the total moment of inertia and the load torque can be calculated from the following equations:

$$J_{\Sigma} = j_{mot} + \sum_{i=2}^{n} \frac{J_i}{i_{1i}^2} + \sum_{j=1}^{k} m_j \, . \rho_{1j}^2 \qquad (16)$$

$$T_r = \sum_{i=1}^{n} \frac{T_i}{i_{1i}} + \sum_{j=1}^{k} F_j \, . \rho_{1j} \qquad (17)$$

N: he number of torques acting on the rotating masses.
K: the number of forces acting on the masses in translational motion.

2.4 Mathematical modelling:

For the mathematical formulation of the dynamic behavior of industrial mechanisms, the most general form is obtained using the Lagrange equations (equation of motions in generalized coordinates) as follows:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = Q_i \qquad (18)$$

$$\mathcal{L} = E_c - E_p b \qquad (19)$$

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$$Q_i = \frac{\delta A_i}{\delta q_i} \qquad (20)$$

Where:
$\mathcal{L}$: Lagrange function,
E_k: Kinetic energy,
E_p:Potential energy,
q_i:Generalized coordinates,
$\dot{q}_i$:: Generalized speeds,
δA_i:: Elementary work,
Q_i:Generalized forces.

Lagrange equations provide a simple method for the mathematical modeling of complex dynamic processes.
The number of equations is determined by the number of the freedom degrees of the system.

2.4.1. Model with three masses;
In the elastic system with three masses, the generalized coordinates are the angular displacements φ_1,φ_2 and φ_(3) of the masses, which are referred to the speeds $\omega_1, \omega_2$and ω_3 respectively.
To determine, for example, the generalized force Q_1, all the torques applied to the first mass with the possible displacement $\delta\varphi_1$ (1)are related as follows:

$$\delta A_1 = \delta\varphi_1 (T - T_{r1})$$

Then,

$$Q_1 = T - T_{r1} \qquad (21)$$

Similarly, for obtaining Q_2 and Q_3.

$$Q_2 = -T_{r2}, \; Q_3 = -T_{r3} \qquad (22)$$

$$E_k = \frac{J_1\omega_1^2}{2} + \frac{J_2\omega_2^2}{2} + \frac{J_3\omega_3^2}{2} \qquad (23)$$

$$E_p = \frac{h_{12}(\varphi_1-\varphi_2)^2}{2} + \frac{h_{23}(\varphi_2-\varphi_3)^2}{2} \qquad (24)$$

If equations (21), (22), (23) and (24) are substituted in the general equation of Lagrange, we obtain the following motion equations of the system may be obtained:

$$\begin{cases} T - h_{12}(\varphi_1 - \varphi_2) - T_{r1} = J_1 \cdot \frac{d\omega_1}{dt} \\ h_{12}(\varphi_1 - \varphi_2) - h_{23}(\varphi_2 - \varphi_3) - T_{r2} = J_2 \cdot \frac{d\omega_2}{dt} \\ h_{23}(\varphi_2 - \varphi_3) - T_{r3} = J_3 \cdot \frac{d\omega_3}{dt} \end{cases} \qquad (25)$$

Where the torques of elastic interactions are:

$$T_{12} = h_{12}(\varphi_1 - \varphi_2)$$
$$T_{23} = h_{23}(\varphi_2 - \varphi_3)$$

This allows expressing the mathematical model of this system in the following form:

$$\begin{cases} T - T_{12} - T_{r1} = J_1 \cdot \frac{d\omega_1}{dt} \\ T_{12} - T_{23} - T_{r2} = J_2 \cdot \frac{d\omega_2}{dt} \\ T_{23} - T_{r3} = J_3 \cdot \frac{d\omega_3}{dt} \end{cases} \qquad (26)$$

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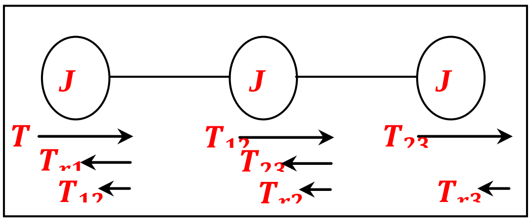

Figure 4: Equivalent computational scheme with 3 masses.

2.4.2 Model with two masses;

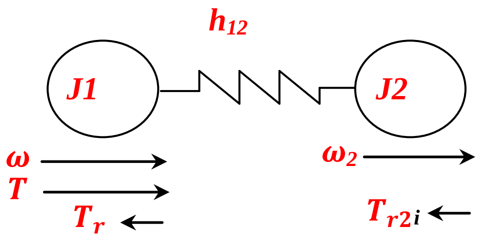

Figure 5: Model of the system with two masses.

In this case, the generalized coordinates are the displacements φ_1, φ_2, and the generalized forces are:

$$Q_1 = \frac{\delta A_1}{\delta \varphi_1} = T - T_{r1} \qquad (27)$$

$$Q_2 = \frac{\delta A_2}{\delta \varphi_2} = -T_{r2} \qquad (28)$$

Using the Lagrange equation, the following equations can be obtained:

$$\begin{cases} T - h_{12}(\varphi_1 - \varphi_2) - T_{r1} = J_1 \cdot \frac{d\omega_1}{dt} \\ h_{12}(\varphi_1 - \varphi_2) - T_{r2} = J_2 \cdot \frac{d\omega_2}{dt} \end{cases} \qquad (29)$$

Substitution of $T_{12} = h_{12}(\varphi_1 - \varphi_2)$ into Eq. (29) gives:

$$\begin{cases} T - T_{12} - T_{r1} = J_1 \cdot \frac{d\omega_1}{dt} \\ T_{12} - T_{r2} = J_2 \cdot \frac{d\omega_2}{dt} \end{cases} \qquad (30)$$

2.4.3 Model of the system with one mass:

In this case, the system is considered as absolutely rigid. Starting from the equivalent circuit with two masses as shown in Figure 5 and by considering that: $h_{12} \approx \infty$, this leads to make the presentation as shown in Figure 6.

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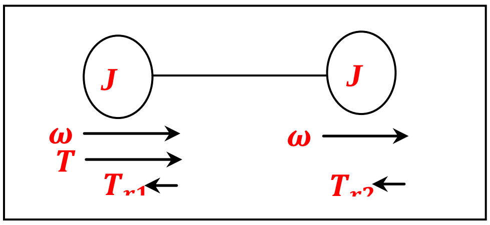

Figure 6: Equivalent scheme of the system with two masses.

Since the system is absolutely rigid, the two masses have the same speed.

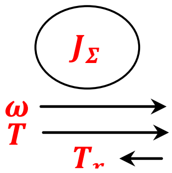

Figure 7: Equivalent scheme of the system with one mass.

By letting $\omega_1 = \omega_2 = \omega$ and summing the terms of the Equ. (26), the following equation can be obtained:

$$T - T_r = J_\Sigma \cdot \frac{d\omega}{dt} \qquad (31)$$

with:
$T_r = T_{r1} + T_{r2}$: Represents the total reverse torque.
$J_\Sigma = J_1 + J_2$: Represents the total moment of inertia.

These models are valid for various applications starting from a simple mechanism with only one degree of freedom to the complex manipulator arms of the robots.

III.CONCLUSION

In this work, we have presented a comprehensive approach allowing the development of simplified computational models for any type of mechanism (rigid or elastic) and its degree of complexity as well as the mathematical formulation of the dynamic behavior of the mechanism.
These simplified models can be easily solved numerically and their solutions can be used for sizing and selection in the design phase. The developed equations of motion of this paper are the essential elements for modelling any electromechanical system. If the equations of dynamic behavior are well known, the study and sizing of electric actuators can be carried out efficiently.

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