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Solving Fuzzy Differential Equations Using Runge-Kutta Third Order Method for Three Stages Contra-Harmonic Mean

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ABSTRACT: In this paper, a study has been carried out on the application of numerical method for solving the first order fuzzy differential equations using Runge-kutta third order method for three stages of contra-harmonic mean. The applicability of the method has been demonstrated by an example and the convergence of the proposed method has been studied.

KEYWORDS: Fuzzy Differential Equations, Runge-kutta third order,Contra-harmonic Mean,Triangular Fuzzy Number

I. INTRODUCTION

The theory of fuzzy differential equation plays an important role in modelling of science and engineering problems because, this theory represents a natural way to model dynamical systems under uncertainty. The applicability of the fuzzy differential equation leads to a several number of research works in the open literature. First order linear fuzzy differential equation is one of the simplest fuzzy differential equation, which appear in many applications. Some of the reviewed research papers are cited below for better understanding of the present paper. The concept of fuzzy derivative was first introduced by S.L.Chang and L.A.Zadeh in[6].D.Dubois and Prade [7] discussed differentiation with fuzzy features.M.L.puri,D.A.Ralescu[24] and R.Goetschel,W.Voxman[10] contributed towards the differential of fuzzy functions. The fuzzy differential equation and initial value problems were extensively studied by O.Kaleva[15,16] and by S.Seikkala[25].Recently many research papers are focused on numerical solution of fuzzy initial value problems (FIVP).Numerical Solution of fuzzy differential equations has been introduced by M.Ma, M.Friedman, A.Kandel [19] through Euler method and by S.Abbasbandy,T.Allahviranloo [1] by taylor method. Runge –Kutta methods have also been studied by authors [2,22].V.Nirmala,N.Saveetha,S.Chenthurpandiyan discussed on numerical Solution of fuzzy differential Equations by Runge-Kutta method with higher order derivative approximations[21]. R.Gethsi sharmila and E.C.Henry Amirtharaj discussed on numerical Solutions of first order fuzzy initial value problems by non-linear trapezoidal formulae based on variety of means[13]. Runge-kutta third order method with Contra-Harmonic mean was discussed by Osama Yusuf Ababneh,Rokiah Rozita[17].

Following by the introduction in section 1 this paper is organised as follows :In section 2, some basic results of fuzzy numbers and definitions of fuzzy derivative are given. In section 3, the fuzzy initial value problem is discussed. Section 4 describes the structure of Runge-kutta third order method for three stages of contra-harmonic mean was proposed. In section 5, the proposed method is used for solving fuzzy differential equations and the numerical example is given to illustrate the applicability of the proposed method followed by, the conclusion given in the last section.

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II. PRELIMINARIES

2.1 FUZZY NUMBER

An arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(r),\overline{u}(r))$ for all $r\in[0,1]$ which satisfy the following conditions.

i) $\underline{u}(r)$ is a bounded left continuous non-decreasing function over $[0,1]$ with respect to any r.

ii) $\overline{u}(r)$ is a bounded right continuous non-decreasing function over $[0,1]$ with respect to any r.

iii) $(\underline{u}(r)\le\overline{u}(r))$ for all $r\in[0,1]$ then the r-level set is $[u]_r=\{x\backslash u(x)\ge r\};0\le r\le 1$

Clearly, $[u]_0=\{x\backslash u(x)\ge 0\}$ is compact, which is a closed bounded interval and we denote by $[u]_r=(\underline{u}(r),\overline{u}(r))$

2.2 TRIANGULAR FUZZY NUMBER

A triangular fuzzy number *u* is a fuzzy set in E that is characterized by an ordered triple $(u_l,u_c,u_r)\in R^3$ with $u_l\le u_c\le u_r$ such that $[u]_0=[u_l;u_r]$ and $[u]_l=\{u_c\}$.

The membership function of the triangular fuzzy number *u* is given by

$$u(x)=\begin{cases}\dfrac{x-u_l}{u_c-u_l} & ; \quad u_l\le x\le u_c\\ 1 & ; \quad x=u_c\\ \dfrac{u_r-x}{u_r-u_c} & ; \quad u_c\le x\le u_r\end{cases}$$

. we have :*(1)* $u>0\ if\ u_l>\ 0$

(2) $u\ge 0\ if\ u_l\ge 0$

(3) $u<0\ if\ u_c<0$ and

(4) $u\le 0\ if\ u_c\le 0$.

2.3: α - Level Set

Let I be the real interval. A mapping $y:I\to E$ is called a fuzzy process and its α - *level Set* is denoted by $[y(t)]_\propto=[\underline{y}(t;\alpha),\overline{y}(t;\alpha)]$, $t\in I, 0<\propto<1$

2.4:Seikkala Derivative

The Seikkala derivative $y'(t)$ of a fuzzy process is defined by $[y'(t)]_\propto=[\underline{y}'(t;\alpha),\overline{y}'(t;\alpha)]$ $t\in I, 0<\propto\le 1$ provided that this equation defines a fuzzy number, as in [25]

2.5:Lemma:

If the sequence of non-negative number $\{W_n\}_{n=0}^{m}$ satisfy $|Wn+1|\le A|Wn|+B$, $0\le n\le N-1$ for the given positive constants *A* and *B*, then $|Wn|\le A^n|W_0|+B\dfrac{A^n-1}{A-1}$, $0\le n\le N$

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2.6:Lemma:

If the sequence of non-negative numbers $\{W_n\}_{n=0}^{m}, \{V_n\}_{n=0}^{N}$ satisfy

$|W_{n+1}| \le |W_n| + A\, max\{|W_n|, |V_n|\} + B,$

$|V_{n+1}| \le |V_n| + A\, max\{|W_n|, |V_n|\} + B$

for the given positive constants *A* and *B*, then $U_n = |W_n| + |V_n|$, $0 \le n \le N$

we have, $U_n \le \overline{A^n} U_0 + B \dfrac{\overline{A^n} - 1}{A - 1}$ $0 \le n \le N$ where $\overline{A} = 1 + 2A$ and $\overline{B} = 2B$.

2.7:Lemma

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F .Then for arbitrarily fixed *r*, $0 \le r \le 1$, $D(y(t_{n+1}), y^0(t_{n+1})) \le h^2 L(1+2C)$ where *L* is a bound of partial derivatives of *F* and *G*, and $C = Max\left\{\left|G\left[t_N, \underline{y}(t_N;r), \overline{y}(t_{N-1};r)\right]\right|, r \in [0,1]\right\} < \infty$

2.8:Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F .Then for arbitrarily fixed *r*, $0 \le r \le 1$ the numerical solutions of $\underline{y}(t_{n+1};r)$ and $\overline{y}(t_{n+1};r)$ converge to the exact solutions $\underline{Y}(t_{n+1};r)$ and $\overline{Y}(t_{n+1};r)$ uniformly in *t*.

2.9:Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C'(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F and $2Lh < 1$. Then for arbitrarily fixed $0 \le r \le 1$,the iterative numerical solutions of $\underline{y}^{(j)}(t_n;r)$ and $\overline{y}^{(j)}(t_n;r)$ converge to the numerical solutions $\underline{y}(t_n;r)$ and $\overline{y}(t_n;r)$ in $t_0 \le t_n \le t_N$, when $j \to \infty$.

III.FUZZY INITIAL VALUE PROBLEM

Consider a first-order fuzzy initial value differential equation is given by

$$\begin{cases} y'(t) = f(t, y(t)), t \in [t_0, T] \\ \quad y(t_0) = y_0 \end{cases} \tag{3.1}$$

where *y* is a fuzzy function of t, $f(t,y)$ is a fuzzy function of the crisp variable 't' and the fuzzy variable y, y' is the fuzzy derivative of *y* and $y(t_0) = y_0$ is a triangular or a triangular shaped fuzzy number.

We denote the fuzzy function *y* by $y = [\underline{y}, \overline{y}]$ .It means that the r-level set of $y(t)$ for $t \in [t_0, T]$ is $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$, $[y(t_0)]_r = [\underline{y}(t_0;\ r), \overline{y}(t_0;\ r)]$, $r \in (0,1]$,

we write $f(t,y) = [\underline{f}(t,y), \overline{f}(t,y)]$ and

$\underline{f}(t,y) = F[t, \underline{y}, \overline{y}]$, $\overline{f}(t,y) = G[t, \underline{y}, \overline{y}]$,

because of $y' = f(t,y)$ we have

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$$\underline{f}\left(t, y(t); r\right) = F[t, \underline{y}(t;r), \overline{y}(t;r)] \tag{3.2}$$

$$\overline{f}\left(t, y(t); r\right) = G[t, \underline{y}(t;r), \overline{y}(t;r)] \tag{3.3}$$

by using the extension principle, we have the membership function

$$f(t, y(t))(s) = sup\{y(t)(\tau) \setminus s = f(t,\tau)\},\ s \in R \tag{3.4}$$

so the fuzzy number $f\left(t, y(t)\right)$ follows that

$$[\,f(t, y(t))]_r = [\,\underline{f}(t, y(t);r), \overline{f}(t, y(t);r)],\ r \in (0,1] \tag{3.5}$$

where $\underline{f}\ (t, y(t); r) =\ min\{\,f(t,u)\,|\,u\left[y(t)\right]_r\}$ (3.6)

$$\overline{f}\left(t, y(t); r\right) = max\{\,f(t,u)\,|\,u\left[y(t)\right]_r\} \tag{3.7}$$

Definition 3.1 A function $f : R \to R_F$ is said to be fuzzy continuous function, if for an arbitrary fixed $t_0 \in R$ and $\varepsilon > 0$, $\delta > 0$ such that $\left|t - t_o\right| < \delta \Rightarrow D\left[\,f(t), f(t_0)\right] < \varepsilon$ exists.

The fuzzy function considered are continuous in metric D and the continuity of $f(t, y(t); r)$ guarantees the existence of the definition of $f(t, y(t); r)$ for $t \in \left[t_0, T\right]$ and $r \in \left[0,1\right]$ [10]. Therefore, the functions *G* and *F* can be definite too.

IV. RUNGE-KUTTA THIRD ORDER METHOD FOR THREE STAGES CONTRA-HARMONIC MEAN

The Runge-kutta third order method for three Stages Contra-harmonic mean for approximating the solution of first order fuzzy initial value problem $y'(t) = f(t, y(t))\ \ y(t_0) = y_0$.

The basis of all Runge-Kutta methods is to express the difference between the value of y at t_{n+1} and t_n as

$$y_{n+1} - y_n = \sum_{i=0}^{m} w_i k_i \tag{4.1}$$

Where w_i 's are constant for all i and $k_i = hf(t_n + a_i h, y_n + \sum_{j=1}^{i-1} c_{ij} k_j)$ (4.2)

Increasing of the order of accuracy of the Runge-Kutta methods it have been accomplished by increasing the number of taylor's series terms used and thus the number of functional evaluations required[5].The method proposed by Goeken.D and Johnson.O[9] introduces new terms involving higher order derivatives of 'f' in the Runge-Kutta k_i terms(i > 0) to obtain a higher order of accuracy without a corresponding increase in evaluations of 'f ', but with the addition of evaluations of f'.

Runge-kutta third order method for three stages Contra-harmonic mean was discussed by Osama Yusuf Ababneh, and Rokiah Rozita [17]

Consider $$y(t_{n+1}) = y(t_n) + \frac{h}{2}\left[\frac{k_1^{\,2} + k_2^{\,2}}{k_1 + k_2} + \frac{k_2^{\,2} + k_3^{\,2}}{k_2 + k_3}\right] \tag{4.3}$$

Where $k_1 = hf(t_n, y(t_n))$ (4.4)

$$k_2 = hf(t_n + a_1, y(t_n) + a_1 k_1) \tag{4.5}$$

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$$k_3 = hf(t_n + a_2, y(t_n) + a_2 k_2) \quad (4.6)$$

and the parameters a_1, a_2 are chosen to make y_{n+1} closer to $y(t_{n+1})$. The value of parameters are $a_1 = \frac{2}{3}, a_2 = \frac{2}{3}$

V. SOLVING FUZZY DIFFERENTIAL EQUATIONS USING RUNGE-KUTTA THIRD ORDER METHOD FOR THREE STAGE CONTRA-HARMONIC MEAN

Let the exact solution $[Y(t)]_r = [\underline{Y}(t;\ r), \overline{Y}(t;\ r)]$, is approximated by some $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$. The grid points at which the solutions is calculated are $h = \frac{T - t_0}{N}$, $t_i = t_0 + ih;\ 0 \le\ i \le\ N$

From 4.3 to 4.6 we define

$$\underline{y}(t_{n+1}, r) - \underline{y}(t_n, r) = \frac{h}{2}\left[\frac{\underline{k_1}^2(t_n, y(t_n,r)) + \underline{k_2}^2(t_n, y(t_n,r))}{\underline{k_1}(t_n, y(t_n,r)) + \underline{k_2}(t_n, y(t_n,r))} + \frac{\underline{k_2}^2(t_n, y(t_n,r)) + \underline{k_3}^2(t_n, y(t_n,r))}{\underline{k_2}(t_n, y(t_n,r)) + \underline{k_3}(t_n, y(t_n,r))}\right] \quad (5.1)$$

where

$$k_1 = hF[t_n, \underline{y}(t_n,r), \overline{y}(t_n,r)] \quad (5.2)$$

$$k_2 = hF[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_1}(t_n, y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_1}(t_n, y(t_n,r))] \quad (5.3)$$

$$k_3 = hF[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_2}(t_n, y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_2}(t_n, y(t_n,r))] \quad (5.4)$$

and

$$\overline{y}(t_{n+1}, r) - \overline{y}(t_n, r) = \frac{h}{2}\left[\frac{\overline{k_1}^2(t_n, y(t_n,r)) + \overline{k_2}^2(t_n, y(t_n,r))}{\overline{k_1}(t_n, y(t_n,r)) + \overline{k_2}(t_n, y(t_n,r))} + \frac{\overline{k_2}^2(t_n, y(t_n,r)) + \overline{k_3}^2(t_n, y(t_n,r))}{\overline{k_2}(t_n, y(t_n,r)) + \overline{k_3}(t_n, y(t_n,r))}\right] \quad (5.5)$$

where

$$k_1 = hG[t_n, \underline{y}(t_n,r), \overline{y}(t_n,r)] \quad (5.6)$$

$$k_2 = hG[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_1}(t_n, y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_1}(t_n, y(t_n,r))] \quad (5.7)$$

$$k_3 = hG[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_2}(t_n, y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_2}(t_n, y(t_n,r))] \quad (5.8)$$

We define

$$F(t_n, y(t_n,r)) = \frac{h}{2}\left[\frac{\underline{k_1}^2(t_n, y(t_n,r)) + \underline{k_2}^2(t_n, y(t_n,r))}{\underline{k_1}(t_n, y(t_n,r)) + \underline{k_2}(t_n, y(t_n,r))} + \frac{\underline{k_2}^2(t_n, y(t_n,r)) + \underline{k_3}^2(t_n, y(t_n,r))}{\underline{k_2}(t_n, y(t_n,r)) + \underline{k_3}(t_n, y(t_n,r))}\right] \quad (5.9)$$

$$G(t_n, y(t_n,r)) = \frac{h}{2}\left[\frac{\overline{k_1}^2(t_n, y(t_n,r)) + \overline{k_2}^2(t_n, y(t_n,r))}{\overline{k_1}(t_n, y(t_n,r)) + \overline{k_2}(t_n, y(t_n,r))} + \frac{\overline{k_2}^2(t_n, y(t_n,r)) + \overline{k_3}^2(t_n, y(t_n,r))}{\overline{k_2}(t_n, y(t_n,r)) + \overline{k_3}(t_n, y(t_n,r))}\right] \quad (5.10)$$

Therefore we have

$$\underline{Y}(t_{n+1}, r) = \underline{Y}(t_n, r) + F[t_n, Y(t_n, r)]$$
$$\overline{Y}(t_{n+1}, r) = \overline{Y}(t_n, r) + G[t_n, Y(t_n, r)] \quad (5.11)$$

and

$$\underline{y}(t_{n+1}, r) = \underline{y}(t_n, r) + F[t_n, y(t_n, r)] \quad (5.12)$$
$$\overline{y}(t_{n+1}, r) = \overline{y}(t_n, r) + G[t_n, y(t_n, r)]$$

Clearly $\underline{y}(t;r)$ and $\overline{y}(t;r)$ converge to $\underline{Y}(t;r)$ and $\overline{Y}(t;r)$ whenever $h \to 0$

VI. NUMERICAL EXAMPLE

Consider a fuzzy initial value problem

$$\begin{cases} y'(t) = y(t), & t \geq 0 \\ y(0) = (0.75 + 0.25r, 1.125 - 0.125r) \end{cases} \quad (6.1)$$

The exact solution is given by

$$Y(t,r) = [(0.75 + 0.25r)e^t, (1.125 - 0.125r)e^t] \quad (6.2)$$

At t=1we get

$$Y(1,r) = [(0.75 + 0.25r)e, (1.125 - 0.125r)e], \; 0 \leq r \leq 1 \quad (6.3)$$

The values of exact and approximate solution with h= 0.1 is given in Table:1.The exact and approximate solutions obtained by the proposed method is plotted in Fig:1 and the error estimation for exact and approximate solution is plotted in Fig:2

Table:1

r	Exact Solution t=1		Approximate Solution (h=0.1)		Error 1	Error 2
	$\underline{Y}(t,r)$	$\overline{Y}(t,r)$	$\underline{y}(t,r)$	$\overline{y}(t,r)$		
0.0	2.038711 ,	3.058067	2.038604 ,	3.057906	1.072212e-004	1.608318e-004
0.1	2.106668 ,	3.024089	2.106558 ,	3.023929	1.107953e-004	1.590448e-004
0.2	2.174625 ,	2.990110	2.174511 ,	2.989953	1.143693e-004	1.572578e-004
0.3	2.242583 ,	2.956131	2.242465 ,	2.955976	1.179433e-004	1.554708e-004
0.4	2.310540 ,	2.922153	2.310418 ,	2.921999	1.215174e-004	1.536837e-004
0.5	2.378497 ,	2.888174	2.378372 ,	2.888023	1.250914e-004	1.518967e-004
0.6	2.446454 ,	2.854196	2.446325 ,	2.854046	1.286655e-004	1.501097e-004
0.7	2.514411 ,	2.820217	2.514278 ,	2.820069	1.322395e-004	1.483227e-004
0.8	2.582368 ,	2.786239	2.582232 ,	2.786092	1.358135e-004	1.465357e-004
0.9	2.650325 ,	2.752260	2.650185 ,	2.752116	1.393876e-004	1.447486e-004
1.0	2.718282 ,	2.718282	2.718139 ,	2.718139	1.429616e-004	1.429616e-004

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Fig – 1(Exact & Approximate)

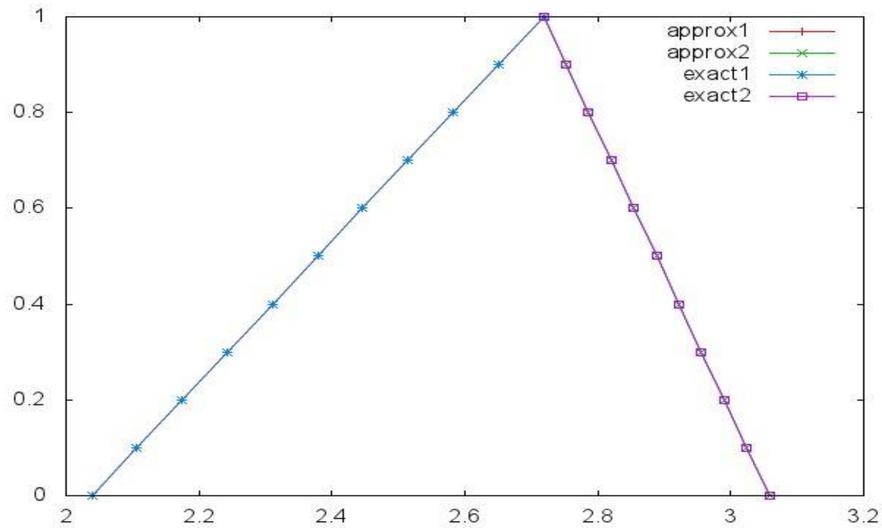
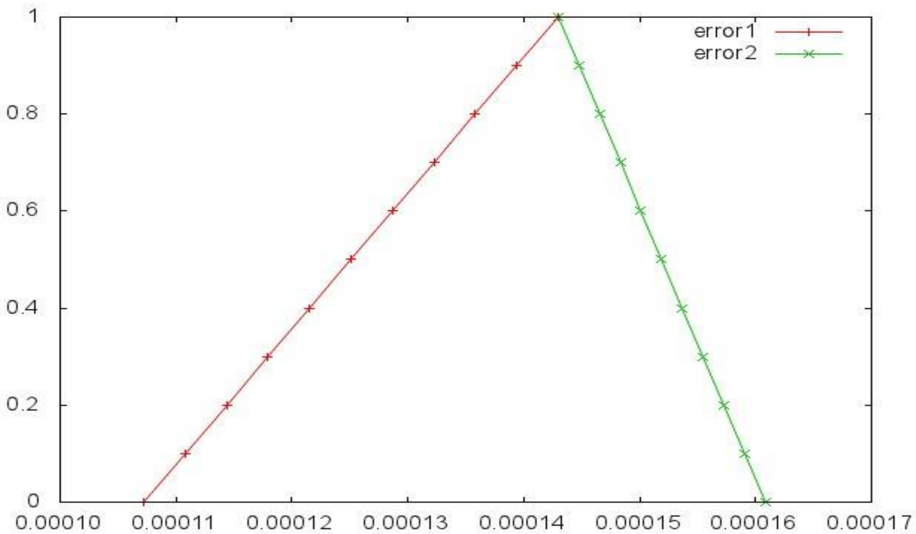


Fig – 2(Error 1 & Error 2)



VII. CONCLUSION

In this paper the runge-kutta third order method for three stage contra-harmonic mean has been applied for finding the numerical solution of first order fuzzy differential equations using triangular fuzzy number. The accuracy and applicability of the proposed method have been illustrated by a suitable example. From the numerical example it has been observed that by minimizing the step size'h' the exact solution at 'h' and the approximate solution obtained by the proposed method almost coincide.

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