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The Stability for Tit-for-Tat

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ABSTRACT

Reciprocity has been a central mechanism which may possibly explain the evolution of cooperation. Tit-for-tat is a famous strategy as a direct reciprocator, and previous studies have pointed out that tit-for-tat is invaded by either unconditional cooperation or unconditional defection. In this study, we comprehensively examine the condition under which tit-for-tat is stable against the invasion by a variety of concrete strategies, and discuss when tit-for-tat is stable and tit-for-tat is not stable.

INTRODUCTION

Direct reciprocity has been an essential mechanism for potentially explaining why animals sometimes behave cooperatively even toward non-kin animals [1]. Direct reciprocators punish defectors by withholding cooperation in the future. Therefore, cooperation can be better off than defection and the evolution of cooperation becomes possible.

Axelrod and Hamilton [2] were conceived tit-for-tat (TFT) strategy as a reciprocal strategy. In the repeated prisoner's dilemma game, the interaction continues on and a player chooses either to cooperate or defect in every round. On one hand, a cooperator confers a benefit b to the other player at a cost c to oneself, where b>c>0 (, which is sometimes called a donation game or a simplified prisoner's dilemma). On the other hand, a defector provides nothing. We denote the probability that in each interaction, any given pair continues on to play the prisoner's dilemma by w, where 0<w<1. Their relationship is broken with probability 1-w. This assumption means that the expected number of interactions is given by 1/(1-w). TFT tries to cooperate in the first move and in the following rounds, tries to cooperate if the opponent player cooperated in the previous round; otherwise, TFT defects. Let us consider the case where players are paired at random.

At first, let us look back the previous studies which considered the case where there are no execution errors. While TFT can be stable against the invasion by unconditional defectors, TFT is not evolutionary stable in the population consisting of TFT, ALLC (unconditional cooperators), and ALLD (unconditional defectors) strategy. In this case, TFT can be invaded by an ALLC mutant since TFT and ALLC are neutral. And then when the frequency of ALLC is over the threshold, an ALLD mutant can invade the population. Thus, TFT is likely to be invaded [3].

Secondly, let us look back the previous studies which considered the case where there execution errors are present [3-8]. We use μ, where 0<μ<1 to denote the probability that execution errors (or mistakes in behavior) occur, i.e., that a player who intends to cooperate actually fails to do so. In addition, we assume that there are no errors in which a player who intends to defect actually fails to do so and cooperates (but see also [9-11] for previous works studying this kind of errors). In this case, TFT is invaded by ALLD when $(c/b-(1-\mu)w)>0$ is met, and TFT is invaded by ALLC when $(c/b-(1-\mu)w)<0$ is met. Thus, one of these two inequalities is met. This means that TFT is invaded by either ALLD or ALLC, irrespective of the cost-to-benefit ratio (see [9,12,13] Brandt and Sigmund, Kurokawa, Nowak and Sigmund for related works).

However, this study examined the case where the intruders are ALLC or ALLD. Our previous study [14] considered the case where an invader is not limited to ALLC or ALLD. Our previous study [14] has found that when TFT is a resident strategy, when a mutant player cooperates "more" than TFT in the game played by the mutant strategy and TFT, $(c/b-(1-\mu)w)>0$ is the stability condition for TFT, while when a mutant player cooperates "less" than TFT in the game played by the mutant strategy and TFT, $(c/b-(1-\mu)w)<0$ is the stability condition for TFT. However, the condition under which TFT is stable against the invasion by a concrete strategy under a concrete condition (μ,w) has not been revealed yet even though this topic is important since TFT can be stable when the cost-to-benefit ratio is higher than the threshold if only strategies by whom TFT is stable when the cost-to-benefit ratio is higher than the threshold against the invasion invade the population of TFTs (similarly, TFT can be stable when the cost-to-benefit ratio is lower than the threshold if only strategies by whom TFT is stable when the cost-to-benefit ratio is lower than the threshold against the invasion invade the population of TFTs). In this paper, we tackle on this problem.

The paper is organized as: in MODEL AND RESULTS, we introduce a variety of mutants and obtain the stability condition for TFT; and in DISCUSSION, we summarize the results and suggest a future work to be undertaken.

MODEL AND RESULTS

In this paper, we consider memory-one strategies, and specify the stability condition for TFT against the memory-one strategies (Note that we do not give the stability condition for TFT against every memory-one strategy because of its difficulty for calculation). We consider the following reactive strategy whose memory is one. The space of strategies for a game for the current case would be a vector of five probabilities: f, P_{CC}, P_{CD}, P_{DC} and P_{DD}. f represents the probability of trying to cooperate in the first round. P_{ij} represents the probability of trying to cooperate when the focal player did i and the opponent did j in the last move, where i is C (cooperation) or D (defection) and j is C or D. f, P_{CC}, P_{CD}, P_{DC} and P_{DD} are not less than 0 and not more than 1 since f, P_{CC}, P_{CD}, P_{DC} and P_{DD} are probabilities.

2-1 The stability condition for TFT when being invaded by famous strategies (e.g., WSLS, grim, STFT, ATFT)

In this section, we obtain the conditions under which TFT is stable against the invasion by several famous strategies.

When an invader is a TFT $(f, P_{CC}, P_{CD}, P_{DC}, P_{DD})= (1,1,0,1,0)$ mutant, the mutant strategy has the same payoff as the payoff the resident strategy gets when a mutant strategy invades a population.

When an invader is an ALLC $(f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1,1,1,1,1)$ mutant, a GTFT mutant (see [15]), a firm but fair $((f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1,1,0,1, P_{DD}))$ mutant [10,16], or a Tweedledee $((f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1,1,0,1,1)$ mutant [17,18] the stability condition for TFT is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)>0 \tag{1}$$

When an invader is an ALLD $((f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (0, 0, 0, 0, 0)$ mutant, a Grim $(f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1, 1, 0, 0, 0)$ mutant [17], or a STFT $(f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (0, 1, 0, 1, 0)$ mutant [19], the stability condition for TFT is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)<0 \tag{2}$$

When an invader is a wins-stay, lose-shift (WSLS) $((f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1, 1, 0, 0, 1))$ mutant [20-26], the stability condition for TFT is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(\mu-\mu^{*})>0 \tag{3}$$

where $\mu^{*}=\dfrac{1-\sqrt{1-w+w^{2}}}{w}$

On one hand, when μ is larger than μ^*, the inequality (3) becomes (1). On the other hand, when μ is smaller than μ^*, the inequality (3) becomes (2).

When an invader is an anti-reciprocation (ATFT) $((f, P_{CC}, P_{CD}, P_{DC}, P_{DD}) = (1, 0, 1, 0, 1))$ mutant [27], the stability condition for TFT is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(\mu-\mu^{**})>0 \tag{4}$$

where $\mu^{**}=\dfrac{1-w}{2-w}$

On one hand, when μ is larger than μ^{**}, the inequality (4) becomes (1). On the other hand, when μ is smaller than μ^{**}, the inequality (4) becomes (2).

Figure 1 illustrates the parameter regions for which TFT is stable against the invasion by a various strategy mutant. When an invader is an ALLC, a GTFT, a firm but fair, or a Tweedledee (or the strategies which cooperate more than TFT in a game played by the strategy and TFT), the stability condition for TFT is (1) (see **Figure 1a**). On the other hand, when an invader is an ALLD, a Grim, or a STFT (or the strategies which cooperate less than TFT in a game played by the strategy and TFT), the stability condition for TFT is (2) (see **Figure 1b**). When a cooperative mutant and a defective mutant invade the population of TFTs at the same time, either of them can successfully invade the population of TFTs (Remember that TFT is subject to the invasion by ALLC or to the invasion by ALLD, as previous studies pointed out). When an invader is WSLS, the stability condition for TFT is (3) (see **Figure 1c1**). When an invader is ATFT, the stability condition for TFT is (4) (see **Figure 1c2**).

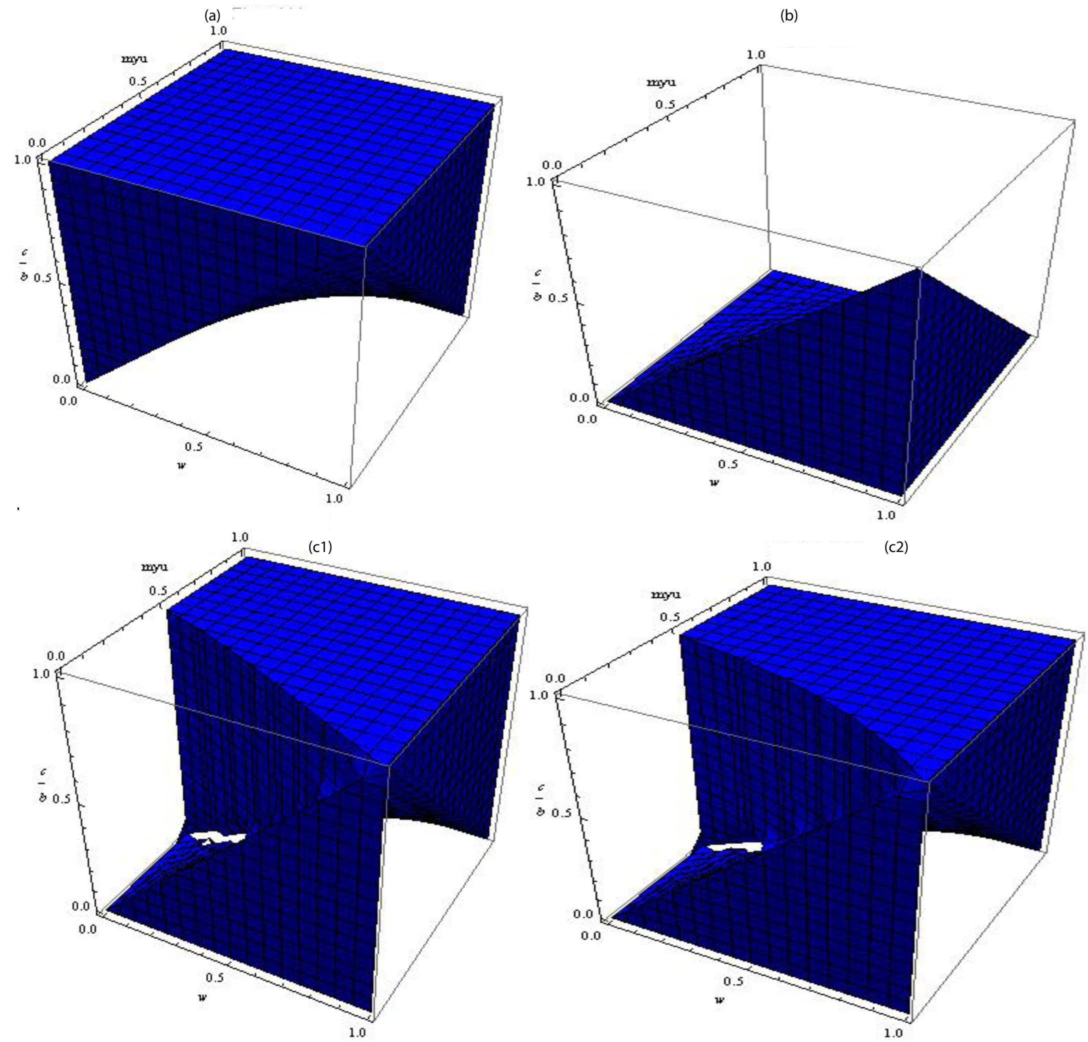

Figure 1. The condition for TFT to be stable against the invasion by a variety of strategy mutants. In the region colored in blue, TFT is an ESS. (a) when the invader is an ALLC, a GTFT, a firm but fair, or a Tweedledee. (b) when the invader is an ALLD, a Grim, or a STFT. (c1) when the invader is a WSLS. (c2) when the invader is an ATFT.

The stability condition for TFT when being invaded by strategies belonging to a more broad strategy set

In the previous section, we examined the conditions under which TFT is stable against the invasion by famous strategies. However, examining the case where other strategies invade the population consisting of TFTs also seems interesting. In the following sections, we examine this.

The case where an invader satisfies $P_{CC}+P_{DD}=P_{CD}+P_{DC}$

We consider the case where a mutant satisfies the constraint $P_{CC}+P_{DD}=P_{CD}+P_{DC}$ as a special case[1,2]. Strategies which do not refer to its own previous action are included in this strategy set since they satisfy the two constraints ($P_{CC}=P_{DC}$ and $P_{DD}=P_{CD}$). ALLC, GTFT, STFT, ALLD, TFT, and ATFT are included in this strategy set. When this reactive strategy with this constraint is an invader, the stability condition for TFT becomes

$$\left(\frac{c}{b}-(1-\mu)w\right)(f-f^{*})>0 \qquad (5)$$

where $f^{*}=\frac{1}{1-(1-\mu)w}-\frac{wP_{DD}}{1-w}-\frac{(1-\mu)w(P_{CC}-P_{DD})}{1-(1-\mu)w}$

When f is larger than the threshold (f^*), (5) becomes (1). On the other hand, when f is smaller than the threshold (f^*), (5) becomes (2). It is also apparent that (5) does not contain either P_{CD} or P_{DC}.

The case where each probability (P_{CC}, P_{DC}, P_{DD}, P_{CD}) for an invader is either 0 or 1

In this section, we remove the constraint $P_{CC}+P_{DD}=P_{CD}+P_{DC}$. Here, we consider the restricted space where f is 1, and each probability (P_{CC}, P_{DC}, P_{DD}, P_{CD}) is either 0 or 1, and the number of strategies which belong to the strategy space is 2^4 (16) [19]. ALLC, Tweedledee, trigger, TFT, ATFT, and WSLS are included in this strategy set. These 16 strategies are classified into the following four groups.

Firstly, we list up the strategies which have the same payoff as the payoff the resident strategy gets when a mutant strategy invades a population. We name the strategy sets Category A. The strategies belonging to Category A are: (1, 1, 1, 0, 0) and (1, 1, 0, 1, 0).

Secondly, we list up the strategies whose stability condition is (1). We name the strategy sets Category B. The strategies belonging to Category B are: (1, 1, 0, 1, 1), (1, 1, 1, 0, 1), (1, 1, 1, 1, 0) and (1, 1, 1, 1, 1).

Thirdly, we list up the strategies whose stability condition is (2). We name the strategy sets Category C. The strategies belonging to Category C are: (1, 1, 0, 0, 0), (1, 0, 1, 0, 0), (1, 0, 0, 1, 0) and (1, 0, 0, 0, 0).

Fourthly, we list up the other strategies. We name the strategy sets Category D. The strategies belonging to Category D are: (1, 1, 0, 0, 1), (1, 0, 1, 1, 0), (1, 0, 0, 0, 1), (1, 0, 1, 1, 1) and (1, 0, 0, 1, 1).

The condition under which the strategy TFT is stable against the invasion by a mutant wins-stay, lose-shift (1, 1, 0, 0, 1) is given by (3), as described above.

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, 1, 0) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(\mu-\mu^{***})>0 \tag{6}$$

where $\mu^{***}=\frac{-1+w+\sqrt{1-w}}{w}$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 0, 0, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(w-w^{*})>0 \tag{7}$$

where $w^{*}=\frac{-3\mu+\mu^{2}+\sqrt{4-8\mu+9\mu^{2}-2\mu^{3}+\mu^{4}}}{2(1-\mu)^{2}}$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, 1, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(w-w^{**})>0 \tag{8}$$

where $w^{**}=\frac{1-3\mu+\mu^{2}}{(1-\mu)^{2}(1+\mu)}$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, 0, 1) is given by (4), as described above.

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 0, 1, 1) is given by (4).

In this section, we considered the case where various mutants invade a population of TFT. Let us observe the strategies belonging to category B. Strategies whose $P_{CC}+P_{CD}+P_{DC}+P_{DD}$ is not less than 3, and whose (P_{CC}, P_{DD}) is not (0, 1) belong to category B. In that case, the mutant strategy cooperates more than TFT in a game between the strategy and TFT in a pair, and the resident strategy is stable when the cost-to-benefit ratio is high. Next, let us observe the strategies belonging to category C. Strategies whose $P_{CC}+P_{CD}+P_{DC}+P_{DD}$ is not more than 1, and whose (P_{CC}, P_{DD}) is not (0, 1) belong to category C. In that case, the mutant strategy cooperates less than TFT in a game between the strategy and TFT in a pair, the resident strategy is stable when the cost-to-benefit ratio is low. Other strategies (i.e., $P_{CC}+P_{CD}+P_{DC}+P_{DD}$ is two or (P_{CC}, P_{DD}) = (0, 1); e.g., WSLS) belong to category A or category D. In that case, the strategies have the same payoff as the payoff the resident strategy gets when a mutant strategy

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invades a population, or whether the resident strategy is stable when the cost-to-benefit ratio is high or low depends on the combination of parameters (μ and *w*).

The case where three probabilities among the four parameters P_{CC}, P_{CD}, P_{DC} and P_{DD} are either 0 or 1, and the rest is more than 0 and but less than 1

In this section, we consider the restricted space where *f* is 1, and three probabilities among the four parameters (P_{CC}, P_{CD}, P_{DC} and P_{DD}) are either 0 or 1, and the rest is more than 0 and but less than 1. Firm but fair is included in this strategy set. In this case, strategies are classified into the following three groups.

Firstly, we list up the strategies whose stability condition is $\left(\frac{c}{b}-(1-\mu)w\right)>0$. The strategies are: (1, 1, 1, 0, P_{DD}), (1, 1, 1, 1, P_{DD}), (1, 1, 0, 1, P_{DD}), (1, 1, 1, P_{DC}, 0), (1, 1, 1, P_{DC}, 1), (1, 1, P_{DC}, 1, 1) and (1, 1, P_{DC}, 1, 0). These strategies cooperate "more" than the strategy TFT in a game between the strategies and TFT in a pair. TFT can be stable against the invasion by these cooperative strategies if the cost-to-benefit ratio is higher than the critical value.

Secondly, we list up the strategies whose stability condition is $\left(\frac{c}{b}-(1-\mu)w\right)<0$. The strategies are: (1, 0, 0, P_{DC}, 0), (1, 1, 0, P_{DC}, 0), (1, 0, P_{DC}, 0, 0), (1, 1, P_{DC}, 0, 0), (1, P_{DC}, 0, 0, 0), (1, P_{DC}, 1, 0, 0), and (1, P_{DC}, 0, 1, 0). These strategies cooperate "less" than the strategy TFT in a game between the strategies and TFT in a pair. TFT can be stable against the invasion by these defective strategies if the cost-to-benefit ratio is smaller than the critical value.

Thirdly, we list up the other strategies. The strategies are: (1, 0, 0, 0, P_{DD}), (1, 0, 0, 1, P_{DD}), (1, 0, 1, 0, P_{DD}), (1, 0, 1, 1, P_{DD}), (1, 1, 0, 0, P_{DD}), (1, 0, 0, P_{DD}), (1, 0, 0, P_{DC}, 1), (1, 0, 1, P_{DC}, 0), (1, 1, 1, P_{DC}, 1), (1, 1, 0, P_{DC}, 1), (1, 0, P_{CD}, 0, 1), (1, 0, P_{CD}, 1, 0), (1, 0, P_{CD}, 1, 1), (1, 1, P_{CD}, 0, 1), (1, P_{CC}, 0, 0, 1), (1, P_{CC}, 0, 1, 1), (1, P_{CC}, 1, 0, 1), (1, P_{CC}, 1, 1, 0) , and (1, P_{CC}, 1, 1, 1). These strategies cooperate "more" in some case and "less" in some case than the strategy TFT when in a game between the strategies and TFT in a pair.

The condition under which the strategy TFT is stable against the invasion by these mutants is given respectively as follows.

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 0, 0, P_{DD}) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)((1-\mu)(1-w)w+P_{DD}w(-\mu^2+w-4\mu w+4\mu^2w-\mu^3w-w^2+3\mu w^2-3\mu^2w^2+\mu^3w^2))<0 \quad (9)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 0, 1, P_{DD}) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)((1-\mu)^2(1-w)(1-w(1-\mu))+P_{DD}(-\mu^2+w-4\mu w+4\mu^2w-\mu^3w-w^2+3\mu w^2-3\mu^2w^2+\mu^3w^2))<0 \quad (10)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, 0, P_{DD}) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-(1-\mu)^2(1-w)+P_{DD}\left(-(1-w)w-\mu^3w(-2+w)-\mu w(-5+3w)+\mu^2\left(1-6w+3w^2\right)\right)\right)>0 \quad (11)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, 1, P_{DD}) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)(-(1-\mu)(1-w)(-1+2\mu(1-w)+w+\mu^2w)+P_{DD}(\mu^3(-2+w)w+(1-w)w+\mu w(-5+3w)+\mu^2(-1+6w-3w^2)))<0 \quad (12)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 1, 0, 0, P_{DD}) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)((1-w)(1-\mu)+P_{DD}(-\mu-\mu w+\mu^2w))<0 \quad (13)$$

+The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 0, P_{DC}, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(\left(1-\mu-\mu^2-3\mu w+4\mu^2w-\mu^3w-w^2+3\mu w^2-3\mu^2w^2+\mu^3w^2\right)-P_{DC}(1-\mu)\left(\mu+w-3\mu w+\mu^2w-w^2+2\mu w^2-\mu^2w^2\right)\right)<0 \quad (14)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, P_{DC}, 0) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left((1-\mu)w+P_{DC}w\left(-\mu-w+2\mu w-\mu^2w\right)\right)<0 \quad (15)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, 1, P_{DC}, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-(1-2\mu-w+\mu w)\left(-1-w+2\mu w-\mu^2w\right)-P_{DC}(1-\mu)\left(\mu+w-3\mu w+\mu^2w-w^2+2\mu w^2-\mu^2w^2\right)\right)<0 \quad (16)$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 1, 0, P_{DC}, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-\mu w\left(1-2\mu-w+\mu^2 w\right)+P_{DC}(1-\mu)\mu w(1-w)\right)>0 \tag{17}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, P_{CD}, 0, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w\left(1-\mu-\mu^2-3\mu w+4\mu^2 w-\mu^3 w-w^2+3\mu w^2-3\mu^2 w^2+\mu^3 w^2\right)-P_{CD}(1-\mu)w\left(-\mu+\mu^2 w\right)\right)>0 \tag{18}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, P_{CD}, 1, 0) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left((1-\mu)w\left(-1+w(1-\mu)\right)+P_{CD}w\mu\right)>0 \tag{19}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 0, P_{CD}, 1, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w(1-2\mu-w+\mu w)-P_{CD}(1-\mu)w\left(-\mu+\mu^2 w\right)\right)>0 \tag{20}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, 1, P_{CD}, 0, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(\mu w\left(1-2\mu-w+\mu^2 w\right)-P_{CD}(1-\mu)\mu w(1-\mu w)\right)<0 \tag{21}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, P_{CC}, 0, 0, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w\left(1-\mu-\mu^2-3\mu w+4\mu^2 w-\mu^3 w-w^2+3\mu w^2-3\mu^2 w^2+\mu^3 w^2\right)+P_{CC}(1-\mu)^2 w\left(1-2\mu w-w^2+\mu w^2\right)\right)>0 \tag{22}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, P_{CC}, 0, 1, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w(1-2\mu-w+\mu w)+P_{CC}(1-\mu)^2 w(1-w-\mu w)\right)>0 \tag{23}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, P_{CC}, 1, 0, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w(1-2\mu-w+\mu w)\left(-1-w+2\mu w-\mu^2 w\right)+P_{CC}(1-\mu)^2 w\left(-1+2\mu w+w^2-\mu w^2\right)\right)<0 \tag{24}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, P_{CC}, 1, 1, 0) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(w\left(1-2\mu-w+2\mu w-\mu^2 w\right)+P_{CC}w\left(-1+\mu+w-2\mu w+\mu^2 w\right)\right)<0 \tag{25}$$

The condition under which the strategy TFT is stable against the invasion by a mutant (1, P_{CC}, 1, 1, 1) is given by

$$\left(\frac{c}{b}-(1-\mu)w\right)\left(-w\left(-1+3\mu-\mu^2+w-\mu w-\mu^2 w+\mu^3 w\right)+P_{CC}(1-\mu)^2 w(-1+w+\mu w)\right)<0 \tag{26}$$

DISCUSSION

Our previous study has found that when TFT is a resident strategy, when a mutant player cooperates "more" than TFT in the game played by the mutant strategy and TFT, $\left(\frac{c}{b}-(1-\mu)w\right)>0$ is the stability condition for TFT, while when a mutant player cooperates "less" than TFT in the game played by the mutant strategy and TFT, $\left(\frac{c}{b}-(1-\mu)w\right)<0$ is the stability condition for TFT. In this paper, we obtained the condition under which TFT is stable against the invasion by concrete strategies.

Previous studies [9,12,13] pointed out that TFT is subject to the invasion by ALLC or to the invasion by ALLD. However, this instability is due to that ALLC is a cooperative strategy and ALLD is a defective strategy. If the invaders are only cooperative strategies (e.g., ALLC, GTFT, firm but fair, Tweedledee) or if the invaders are only defective strategies (e.g., ALLD, Grim, STFT), then this is not the case.

This paper made a stability analysis assuming infinite populations. However, in reality, the population size is finite, and in finite populations, the effect of genetic drift is present [28-43]. Regarding the stability analysis in finite populations, further study is required.

FOOTNOTE

1 According to some previous studies [44,45], some animals satisfy $P_{CC}+P_{DD} \cong P_{CD}+P_{DC}$ though the previous studies did not point it out.

2 See Kurokawa [46] for a related work

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